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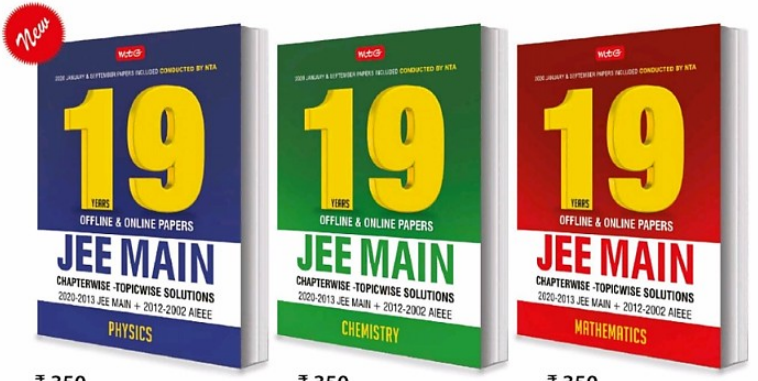

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MATHEMATICS today

Vol. XXXIX No. 1 January 2021

Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurugram -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
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Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
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GEAR UP for JEE MAIN 2021

Exam Dates 2021
23rd to 26th February; 15th to 18th March
27th to 30th April; 24th to 28th May

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION A (MULTIPLE CHOICE QUESTIONS)

1. Let $f(x) = \frac{1}{\sqrt{|x|-x}}$ and $g(x) = -\frac{1}{\sqrt{x-|x|}}$, then
(a) Domain of f and g is ϕ.
(b) Domain of $f = R$ and domain of $g = \phi$.
(c) Domain of f is $(-\infty, 0)$ and domain of g is ϕ.
(d) Domain of f is R and domain of g is $(-\infty, 0)$.

2. A man walks a distance of 3 units from the origin towards the north east (N 45° E) direction. From there he walks a distance of 4 units towards the north – west (N 45° W) direction to a point P. Then the position of P in the Argand plane is
(a) $e^{\frac{i\pi}{4}} + 4i$ (b) $(3-4i)\, e^{\frac{i\pi}{4}}$
(c) $(4+3i)\, e^{\frac{i\pi}{4}}$ (d) $(3+4i)\, e^{\frac{i\pi}{4}}$

3. For natural numbers m, n if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + \ldots$ and $a_1 = a_2 = 10$, then (m, n), is
(a) (20, 45) (b) (35, 20)
(c) (45, 35) (d) (35, 45)

4. A square of side a lies above the x-axis and has one vertex at the origin O. A side passing through the origin makes an angle $\alpha, 0 < \alpha < \frac{\pi}{4}$, with the positive x-axis. The equation of its diagonal not passing through O is
(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$
(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(d) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

5. The directrix of an ellipse with eccentricity $1/\sqrt{2}$ is $x + y = 1$. If the corresponding focus is (1, 2), then the length of the minor axis is
(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

6. The point on $y^2 = 8x$ which is nearest to the circle $x^2 + (y+6)^2 = 1$ is
(a) (2, 4) (b) (2, –4)
(c) (18, –12) (d) (18, 12)

7. Let p : 57 is an odd prime number,
q : 4 is a divisor of 12,
r : 15 is the LCM of 3 and 5
be three simple logical statements. Which one of the following is true?
(a) $p \vee (\sim q \wedge r)$ (b) $\sim p \vee (q \wedge r)$
(c) $(p \wedge q) \vee \sim r$ (d) $(p \vee q) \wedge \sim r$

8. If $\lim_{x\to 0} \frac{a\sin x - bx + cx^2 + x^3}{2x^2 \ln(1+x) - 2x^3 + x^4} = L$, where L is finite and non-zero, then L =
(a) 1/20 (b) 3/20 (c) 3/40 (d) 5/11

9. The mean of the data is 30 and data are given as below.

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	8	10	f_1	15	f_2

If total data is 70, then missing numbers are
(a) 14, 23 (b) 25, 21
(c) 24, 20 (d) 19, 26

10. Let $f : R \to R$ defined by $f(x) = 2x^3 + 2x^2 + 300x + 5\sin x$, then f is
(a) one-one onto (b) one-one into
(c) many one onto (d) many one into

11. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, the number of distinct complex numbers z satisfying $|A| = 0$, where $A = \begin{pmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{pmatrix}$ is

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(a) only three (b) only two
(c) only one (d) more than three

12. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular to each other and the unit vector $\vec{c}$ be inclined at an angle 'θ' to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma(\vec{a} \times \vec{b})$ then,
(a) $\alpha = \beta = \cos\theta, \gamma^2 = \cos 2\theta$
(b) $\alpha = \beta = \cos\theta, \gamma^2 = -\cos 2\theta$
(c) $\alpha = \cos\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$
(d) $\alpha = \sin\theta, \beta = \cos\theta, \gamma^2 = -\cos 2\theta$

13. The image of the point (1, 3, 4) in the plane $x + 2y - z + 3 = 0$ is
(a) (1, 1, – 6) (b) (– 1, – 1, 6)
(c) (– 1, 1, – 6) (d) None of these

14. How many times a person toss a fair coin so that the probability of having atleast one tail is more than 80%?
(a) At least 3 times (b) At most 3 times
(c) at least 5 times (d) at least 6 times

15. Let $f(x) = \begin{cases} xe^{ax}, & x \le 0 \\ x + ax^2 - x^3, & x > 0 \end{cases}$ where $a > 0$ is a constant. The interval in which $f'(x)$ is increasing is
(a) $\left(-\frac{2}{a}, \frac{a}{3}\right)$ (b) $\left(-\frac{3}{a}, \frac{a}{2}\right)$
(c) $\left(\frac{-1}{a}, a\right)$ (d) $\left(-\frac{a}{3}, \frac{2}{a}\right)$

16. If the tangent to the curve $x = 1 - 3t^2, y = t - 3t^3$ at the point $P(-2, 2)$ meets the curve again at Q, then the angle between the tangents at P and Q is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

17. Find the value of the integral $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$.
(a) $\sqrt{\tan x} - \frac{(\tan x)^{\frac{3}{2}}}{3} + c$ (b) $\sqrt{\tan x} + \frac{(\tan x)^{\frac{3}{2}}}{3} + c$
(c) $\sqrt{\tan x} - \frac{(\tan x)^{\frac{5}{2}}}{5} + c$ (d) $\sqrt{\tan x} + \frac{(\tan x)^{\frac{5}{2}}}{5} + c$

18. The area common to the curves $x^2 + y^2 = 2$ and $x^2 + 4y^2 = 4$ is
(a) $\tan^{-1} 2\sqrt{2}$ (b) $2\tan^{-1} 2\sqrt{2}$
(c) $4\tan^{-1}\sqrt{2}$ (d) $4\tan^{-1} 2\sqrt{2}$

19. The temperature of a hot body decreases at a rate proportional to the excess of the temperature of the body over the temperature of the surrounding medium. A body heated to 100° is kept in a room with temperature 30° at time $t = 0$. After 5 minutes the temperature T drops to 80°. Its temperature will be 60° when $t =$
(a) 10 (b) $10 \ln \frac{7}{5}$
(c) 6 (d) $5\log_{7/5} \frac{7}{3}$

20. If $f(x) = ||\sin(|x| - 1)| - 2|$, then
(a) $f(x)$ is continuous but not differentiable at $x = 2$.
(b) $f(x)$ is discontinuous at $x = 2$.
(c) $f(x)$ is differentiable at $x = 2$ and $f'(2) = \cos 1$.
(d) $f(x)$ is differentiable at $x = 2$ and $f'(2) = -\cos 1$.

SECTION B (NUMERICAL VALUE TYPE)

21. The number of roots of the equation $\sin 2x + \cos 2x + \sin x + \cos x + 1 = 0$ in $[0, 2\pi]$ is _____.

22. If $|z| = r$ and z_1, z_2 are the roots of the equation $z^2 + |z|z + \bar{z}z = 0$, then $|z_1 - z_2|^2 = \lambda r^2$, where λ is _____.

23. A man moves one unit distance for each step he takes either along the positive x-direction or negative x-direction. The number of ways he moves from the point (0, 0) to (2, 0) in at most 10 steps is _____.

24. The sum to 20 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \ldots$ is _____.

25. The probability that a teacher will given a unannounced test is 1/4. If a student was absent twice, then probability that he will missed at least one test, is _____.

26. Let A be a 5×5 matrix such that $\det(A) = -3$, then $\det(-3A^{-1}) + 3\det(A)$ equals _____.

27. A swimmer S is in the sea at a distance 4 km from the closest point A on a straight shore. His house B is on the shore at a distance 5 km from A. He can swim at a speed of 3 km per hour and walk at a speed of 5 km per hour. The distance of the point on the shore from B so that he reaches his home in the minimum time is _____.

28. If $\int_0^{\frac{\pi}{3}} \frac{\cos x}{3 + 4\sin x} dx = k \ln\left(\frac{3 + 2\sqrt{3}}{3}\right)$, then $k =$ _____.

29. Let $f(x)$ be a twice differentiable function and $f''(0) = 2$ then $\lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} =$ _____.

30. For $x, y, z, t \in R$ if $\sin^{-1}x + \cos^{-1}y + \sec^{-1}z > t^2 - \sqrt{2\pi}\, t + 3\pi$, then the value of $\tan^{-1}x + \tan^{-1}z + \tan^{-1}\left(\sqrt{\frac{2}{\pi}}t\right)$ is _____.

SOLUTIONS

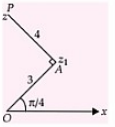
1. **(c):** For domain of f, $|x| > x$ and $\forall$ negative x this is satisfied.

$\Rightarrow$ Domain of $f = (-\infty, 0)$

For domain of g, $x > |x|$ and which is not satisfied for any x

$\Rightarrow$ Domain of $g = \phi$

2. **(d) :** $z_1 = 3\,\text{cis}\,\frac{\pi}{4} \Rightarrow \angle OAP = \frac{\pi}{2}$



AP is obtained by rotating OA about A through $\frac{\pi}{2}$ in the clockwise sense and multiplying by $\frac{4}{3}$

$\therefore\ z - z_1 = (0 - z_1)(-i)\frac{4}{3}$

$$z = z_1\left(1 + \frac{4i}{3}\right) = 3\,\text{cis}\left(\frac{\pi}{4}\right)\cdot\left(1 + \frac{4i}{3}\right)$$

$$= \text{cis}\,\frac{\pi}{4}\cdot(3 + 4i) = (3 + 4i)e^{i\pi/4}$$

3. **(d) :** We have, $(1 - y)^m (1 + y)^n$

$$\left({}^mC_0 - {}^mC_1 y + {}^mC_2 y^2 - \ldots\right)\times\left({}^nC_0 + {}^nC_1 y + {}^nC_2 y^2 + \ldots\right)$$

$$= 1 + (n - m)y + \left\{\frac{m(m-1) + n(n-1)}{2} - mn\right\}y^2 + \ldots$$

$\therefore\quad a_1 = n - m$ and $a_2 = \frac{m(m-1) + n(n-1)}{2} - mn$

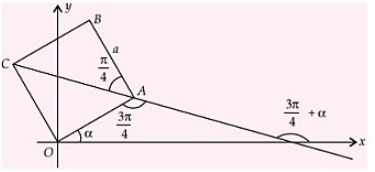
$\Rightarrow\quad n - m = 10$ and $\frac{m^2 + n^2 - (m+n) - 2mn}{2} = 10$

$[\because a_1 = a_2 = 10]$

$\Rightarrow\quad n - m = 10$ and $(m - n)^2 - (m + n) = 20$

$\Rightarrow\quad m - n = -10$ and $m + n = 80 \Rightarrow m = 35$ and $n = 45$

4. **(d) :** AC makes an angle $\frac{3\pi}{4} + \alpha$ with x-axis.



$\therefore\quad$ The slope of $AC = \tan\left(\frac{3\pi}{4} + \alpha\right)$

$$= \tan\left(\alpha - \frac{\pi}{4}\right) = \frac{\tan\alpha - 1}{1 + \tan\alpha} = \frac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}$$

The vertex A is $(a\cos\alpha, a\sin\alpha)$

The equation of AC is

$$\frac{y - a\sin\alpha}{x - a\cos\alpha} = \frac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}$$

or $\quad y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a.$

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

1−	20×		3+	3+	
	3+			24×	5+
2−		12+			
1−	8+			11+	
		2−	3×		9+
9+			2×		

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5. (c): The distance of $S(1, 2)$ from the directrix $x + y = 1$ is $\frac{1+2-1}{\sqrt{2}} = \sqrt{2}$

$\therefore \frac{a}{e} - ae = \sqrt{2} \Rightarrow \sqrt{2}a - \frac{a}{\sqrt{2}} = \sqrt{2} \Rightarrow a = 2$

$2b = 2a\sqrt{1-e^2} = 4\sqrt{1-\frac{1}{2}} = 2\sqrt{2}.$

6. (b) : The shortest distance is along the common normal of the two curves. The normal at t is $xt + y = at^3 + 2at$

$\therefore \quad xt + y = 2t^3 + 4t$, since $a = 2$

A normal to a circle passes through the centre $(0, -6)$.

$\therefore \quad t^3 + 2t + 3 = 0 \Rightarrow t = -1$ and the point is $(a, -2a)$ or $(2, -4)$.

7. (b) : We have, p is false statement, q is true statement and r is also true statement.

Now,

(a) $p \vee (\sim q \wedge r) \equiv F \vee (F \wedge T) \equiv F \vee F \equiv F$

(b) $\sim p \vee (q \wedge r) \equiv T \vee (T \wedge T) \equiv T \vee T \equiv T$

(c) $(p \wedge q) \vee \sim r \equiv (F \wedge T) \vee F \equiv F \vee F \equiv F$

(d) $(p \vee q) \wedge \sim r \equiv (F \vee T) \wedge F \equiv T \wedge F \equiv F$

8. (c): By series, $\lim_{x\to 0} \frac{a\left(x - \frac{x^3}{6} + \frac{x^5}{120} - \ldots\right) - bx + cx^2 + x^3}{2x^2\left(x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\right) - 2x^3 + x^4}$

$= \lim_{x\to 0} \frac{(a-b)x + cx^2 + \left(1 - \frac{a}{6}\right)x^3 + \frac{a}{120}x^5 + \ldots}{\frac{2}{3}x^5 + \ldots} = L$

$\Rightarrow \quad a = b,\ c = 0,\ 1 - \frac{a}{6} = 0 \Rightarrow a = 6$

$\therefore \quad L = \frac{a}{120} \times \frac{3}{2} = \frac{3}{40}.$

9. (a) :

C.I.	x	f	fx
0-10	5	8	40
10-20	15	10	150
20-30	25	f_1	$25f_1$
30-40	35	15	525
40-50	45	f_2	$45f_2$
		$\Sigma f = 70$	$\Sigma fx = 25f_1 + 45f_2 + 715$

Now, $33 + f_1 + f_2 = 70 \Rightarrow f_1 + f_2 = 37$ ...(i)

and $\bar{x} = \frac{\Sigma fx}{\Sigma f} \Rightarrow 30 = \frac{25f_1 + 45f_2 + 715}{70}$

$\Rightarrow \quad 2100 = 25f_1 + 45f_2 + 715$

$\Rightarrow \quad 25f_1 + 45f_2 = 1385$

$\Rightarrow \quad 5f_1 + 9f_2 = 277$ ...(ii)

Solving (i) and (ii), we get $f_1 = 14$, $f_2 = 23$

10. (a) : $f(x) = 2x^3 + 2x^2 + 300x + 5\sin x$

$f'(x) = 6x^2 + 4x + 300 + 5\cos x$

$= 2(3x^2 + 2x + 147) + (6 + 5\cos x)$

Since $-1 \le \cos x \le 1\ \forall\ x \in R$

$\therefore \quad 6 + 5\cos x > 0$

let $g(x) = 3x^2 + 2x + 147$

Since $a = 3 > 0$ and $D = (2)^2 - 4 \times 3 \times 147 < 0$

$\therefore \quad g(x) > 0$

$\therefore \quad f'(x) > 0$

$\therefore \quad f(x)$ is an increasing function therefore said to be one – one function.

Now, $f(-\infty) = -\infty$ and $f(\infty) = \infty$

$\therefore \quad f(x)$ is continuous also.

So, range of f = codomain of $f = R$.

Thus, f is onto.

11. (c): $\because |A| = 0$

$\Rightarrow \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2 + R_3$ and taking z common, we get

$z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$\Rightarrow \quad z = 0$ or

$[(z + \omega^2)(z + \omega) - 1] - 1[\omega(z + \omega) - \omega^2] + 1[\omega - \omega^2(z + \omega^2)] = 0$

$\Rightarrow \quad z = 0$ or $z^2 + z(\omega + \omega^2) - \omega z - \omega^2 z = 0$

$\Rightarrow \quad z = 0$ or $z^2 = 0$

$\therefore \quad$ Only $z = 0$ satisfies the condition $|A| = 0$

$\therefore \quad$ Only $z = 0$ is required solution.

$\therefore \quad$ Required number of complex numbers is only one

12. (b) : $\vec{a}\cdot\vec{b} = 0,\ \vec{a}\cdot\vec{c} = \cos\theta,\ \vec{b}\cdot\vec{c} = \cos\theta$

($\because \vec{a}, \vec{b}, \vec{c}$ are unit vectors)

Also, $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma(\vec{a}\times\vec{b})$

$\therefore \quad \vec{c}\cdot\vec{a} = \alpha + \beta\cdot 0 + \gamma(\vec{a}\times\vec{b})\cdot\vec{a} \Rightarrow \cos\theta = \alpha$

Similarly, $\beta = \cos\theta$

Now, $\gamma(\vec{a}\times\vec{b}) = \vec{c} - \alpha\vec{a} - \beta\vec{b}$

$\Rightarrow \quad |\gamma(\vec{a}\times\vec{b})|^2$

MtG

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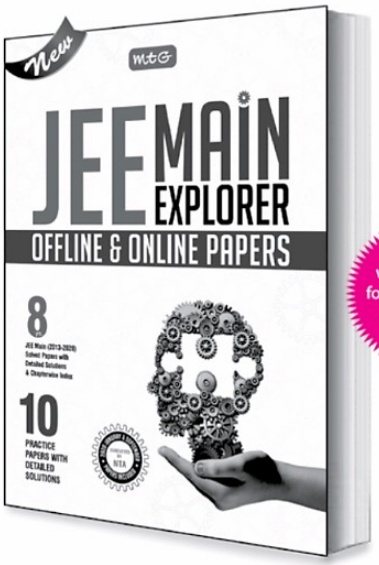

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$= |\vec{c}|^2 + \alpha^2 |\vec{a}|^2 + \beta^2 |\vec{b}|^2 - 2\alpha\vec{a}\cdot\vec{c} - 2\beta\vec{b}\cdot\vec{c}$

$\Rightarrow \quad \gamma^2 = 1 + \alpha^2 + \beta^2 - 2\alpha\cos\theta - 2\beta\cos\theta$

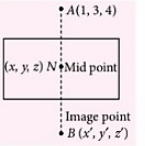
$\Rightarrow \quad \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

13. (b) : Equation of plane is $x + 2y - z + 3 = 0$

So d.r.'s of normal to the plane are $< 1, 2, -1 >$

$\therefore$ Equation of line AB is

$$\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-4}{-1} = \lambda \text{ (say)}$$



Any point on line AB is

$(\lambda + 1, 2\lambda + 3, -\lambda + 4)$

Let $N(x, y, z)$

$= N(\lambda + 1, 2\lambda + 3, -\lambda + 4)$

Using this point in equation of plane, we have

$(\lambda + 1) + 2(2\lambda + 3) - (-\lambda + 4) + 3 = 0$

$\therefore \quad \lambda = -1$

$\therefore$ Midpoint $N(x, y, z) = N(0, 1, 5)$

Now, $\frac{x'+1}{2} = x, \quad \frac{y'+3}{2} = y, \quad \frac{z'+4}{2} = z$

$\therefore \quad \frac{x'+1}{2} = 0 \Rightarrow x' = -1$

Similarly, $y' = -1, z' = 6$

$\therefore$ Required image is $(-1, -1, 6)$

14. (a) : Let the person tosses the coin n times, X denotes the number of tails.

$\therefore \quad P(X \geq 1) > 80\%$

$$\Rightarrow \quad 1 - P(X = 0) > \frac{80}{100} \quad \Rightarrow \quad 1 - {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n > \frac{4}{5}$$

$$\left(\because P(X = r) = {}^nC_r p^r q^{n-r} = {}^nC_r\left(\frac{1}{2}\right)^n\right)$$

$$\Rightarrow \quad 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow \frac{1}{2^n} < \frac{1}{5}$$

$\Rightarrow \quad 2^n > 5$, which can be true only if least value of $n = 3$.

Therefore, coin must be tossed at least 3 times.

15. (a) : $f'(x) = \begin{cases} (1+ax)e^{ax}, & x \leq 0 \\ 1 + 2ax - 3x^2, & x > 0 \end{cases}$

$f''(x) = \begin{cases} (2+ax)ae^{ax}, & x \leq 0 \\ 2a - 6x, & x > 0 \end{cases}$

$f'(x)$ increases $\Rightarrow f''(x) > 0 \Rightarrow 2 + ax > 0, \quad 2a - 6x > 0$

$$\Rightarrow \quad x > \frac{-2}{a}, x < \frac{a}{3} \Rightarrow x \in \left(\frac{-2}{a}, \frac{a}{3}\right)$$

16. (d) : $\frac{dy}{dx} = \frac{9t^2 - 1}{6t}$

$x = -2, y = 2 \Rightarrow t = -1$

Now, $\frac{dy}{dx} = \frac{-4}{3}$ ...(i)

The tangent at P is, $y - 2 = \frac{-4}{3}(x + 2)$

$\Rightarrow \quad 4x + 3y = -2 \Rightarrow 4(1 - 3t^2) + 3(t - 3t^3) = -2$

$\Rightarrow \quad 9t^3 + 12t^2 - 3t - 6 = 0 \Rightarrow 3t^3 + 4t^2 - t - 2 = 0$

$\Rightarrow \quad (t + 1)^2 (3t - 2) = 0$

$\therefore \quad$ At Q, $t = \frac{2}{3}, \frac{dy}{dx} = \frac{9\left(\frac{4}{9}\right) - 1}{6\left(\frac{2}{3}\right)} = \frac{3}{4}$ ...(ii)

From (i) and (ii), we get $\left(\frac{dy}{dx}\right)_P \times \left(\frac{dy}{dx}\right)_Q = -1$

So, the tangents are perpendicular.

17. (d) : $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \frac{1}{2}\int \frac{dx}{(\cos x)^{\frac{7}{2}}\sqrt{\sin x}}$

$= \frac{1}{2}\int \frac{dx}{\cos^4 x\sqrt{\tan x}} = \frac{1}{2}\int \frac{1+t^2}{\sqrt{t}}dt$ [By putting $\tan x = t$]

$= \frac{1}{2}\int \left(t^{-\frac{1}{2}} + t^{\frac{3}{2}}\right)dt = \sqrt{t} + \frac{t^{\frac{5}{2}}}{5} + c = \sqrt{\tan x} + \frac{(\tan x)^{\frac{5}{2}}}{5} + c$

18. (d) : Eliminating y, we get $x^2 + 4(2 - x^2) = 4$

$\Rightarrow x = \pm\frac{2}{\sqrt{3}}$

The shaded area, $I = \int_0^{\frac{2}{\sqrt{3}}} y(\text{ellipse})\,dx + \int_{\frac{2}{\sqrt{3}}}^{\frac{\sqrt{2}}{2}} y(\text{circle})\,dx$

$= \frac{1}{2}\int_0^{\frac{2}{\sqrt{3}}} \sqrt{4 - x^2}\,dx + \int_{\frac{2}{\sqrt{3}}}^{\frac{\sqrt{2}}{2}} \sqrt{2 - x^2}\,dx$

SOLUTION SENDERS

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- Varun Chaturvedi (Gujarat)

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- Sampurnaa Nandi (West Bengal)

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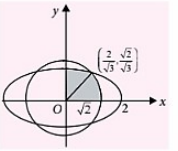
$$= \left|\frac{1}{4}x\sqrt{4-x^2} + \sin^{-1}\frac{x}{2}\right|_0^{\frac{2}{\sqrt{3}}}$$

$$+ \left|\frac{x}{2}\sqrt{2-x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right|_{\frac{2}{\sqrt{3}}}^{\sqrt{2}}$$



$$= \frac{1}{2\sqrt{3}}\sqrt{4-\frac{4}{3}} - \frac{1}{\sqrt{3}}\sqrt{2-\frac{4}{3}} + \sin^{-1}\frac{1}{\sqrt{3}} + \frac{\pi}{2} - \sin^{-1}\frac{\sqrt{2}}{\sqrt{3}}$$

$$= \sin^{-1}\frac{1}{\sqrt{3}} + \cos^{-1}\sqrt{\frac{2}{3}} = 2\tan^{-1}\frac{1}{\sqrt{2}} = \tan^{-1}2\sqrt{2}$$

Required area $= 4I = 4\tan^{-1}2\sqrt{2}$.

19. (d) : $\frac{dT}{dt} = -k(T-30) \Rightarrow \frac{dT}{T-30} = -kdt$

$T - 30 = Ae^{-kt}$

When $t = 0$, $T = 100 \Rightarrow A = 70$

$\therefore \quad T - 30 = 70e^{-kt}$

When $t = 5$, $T = 80 \Rightarrow 5 = 7e^{-5k} \Rightarrow e^{5k} = \frac{7}{5}$

Now, $T = 60 \Rightarrow 30 = 70e^{-kt} \quad \Rightarrow \quad e^{kt} = \frac{7}{3}$

$$\Rightarrow \quad \frac{7}{3} = (e^{5k})^{\frac{t}{5}} = \left(\frac{7}{5}\right)^{\frac{t}{5}} \Rightarrow \frac{t}{5}\log\frac{7}{5} = \log\frac{7}{3}$$

$$\Rightarrow t = 5\log_{(7/5)}\frac{7}{3}.$$

20. (d) : Around $x = 2$, $\sin(|x| - 1) = \sin(x - 1)$
and $\sin(x - 1) > 0 \quad \Rightarrow \quad |\sin(x - 1)| = \sin(x - 1)$
we know, $\sin(x - 1) < 2$
Hence $|\sin(x - 1) - 2| = 2 - \sin(x - 1)$
$f(x) = 2 - \sin(x - 1)$ for $x \in (2 - \delta, 2 + \delta)$
$f'(2) = -\cos 1$

21. (4) : $\sin 2x + (\cos 2x + 1) + \sin x + \cos x = 0$
$\Rightarrow \quad 2\sin x\cos x + 2\cos^2 x + \sin x + \cos x = 0$
$\Rightarrow \quad (\sin x + \cos x)(2\cos x + 1) = 0$
Either $\sin x + \cos x = 0$ or $2\cos x + 1 = 0$
Now, $\sin x + \cos x = 0 \Rightarrow \tan x = -1$
$\therefore \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$
and $2\cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{2} \quad \therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$

22. (3) : $z = re^{i\theta} \Rightarrow e^{2i\theta} + e^{i\theta} + 1 = 0$
$\Rightarrow \quad \cos 2\theta + \cos\theta + 1 = 0, \sin 2\theta + \sin\theta = 0$
$\Rightarrow \quad \theta = \pm\frac{2\pi}{3}, z_1 = re^{\frac{i2\pi}{3}} = r\omega$
$z_2 = re^{\frac{-2\pi i}{3}} = r\omega^2$, where ω is a complex cube root of unity.
$|z_1 - z_2|^2 = r^2|\omega - \omega^2|^2$
$= r^2\left|-\frac{1}{2} + \frac{i\sqrt{3}}{2} + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right|^2 = r^2|i\sqrt{3}|^2 = 3r^2.$
$\therefore \quad \lambda = 3$

23. (286) : Let E and W stand for one unit distance along positive x-direction and negative x-direction respectively.
$EE \quad \rightarrow$ one way
$EEEW \quad \rightarrow \binom{4}{1} = 4$ ways
$EEEEWW \quad \rightarrow \binom{6}{2} = 15$ ways
$EEEEEWWW \rightarrow \binom{8}{3} = 56$ ways
$EEEEEEWWWW \rightarrow \binom{10}{4} = 210$ ways
The desired number of ways is $1 + 4 + 15 + 56 + 210 = 286$.

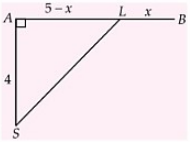
24. (4410): $(1^2 + 2^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$
$= \frac{20\times21\times41}{6} + 4(1^2 + 2^2 + \ldots + 10^2) = 2870 + 1540$
$= 4410.$

25. (0.4375) : The probability that only one test was held in those day in which the student was absent
$= 2 \times \frac{1}{4}\times\frac{3}{4} = \frac{3}{8}$
Now, the probability that the test was held on both the days $= \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$
$\therefore \quad$ The probability that student missed at least one test $= \frac{3}{8} + \frac{1}{16} = \frac{7}{16} = 0.4375$

26. (72) : As $\det(A) = -3 \neq 0$
$\therefore \quad A^{-1}$ exists and the order of A^{-1} will be also 5×5.
Now, $\det(-3A^{-1}) + 3\det(A)$
$= (-3)^5\det(A^{-1}) + 3\times(-3) = (-3)^5(\det(A))^{-1} - 9$
$= (-3)^5\cdot\frac{1}{\det(A)} - 3^2 = (-3)^5\left(-\frac{1}{3}\right) - 3^2$
$= 3^2(3^2 - 1) = 72$

27. (2) : T = Total time

$$= \frac{SL}{3} + \frac{LB}{5}$$



$$= \frac{\sqrt{(5-x)^2+16}}{3} + \frac{x}{5}$$

$$= \frac{\sqrt{x^2-10x+41}}{3} + \frac{x}{5}$$

Now, $\frac{dT}{dx} = \frac{x-5}{3\sqrt{x^2-10x+41}} + \frac{1}{5} = 0$

$\Rightarrow \quad x^2 - 10x + 16 = 0 \Rightarrow x = 2.$

28. (0.25) : Let $I = \frac{1}{4}\int_0^{\frac{\pi}{3}} \frac{4\cos x}{3+4\sin x}dx$

$$= \frac{1}{4}\left[\ln(3+4\sin x)\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{4}\left[\ln\left(3+2\sqrt{3}\right) - \ln 3\right] = \frac{1}{4}\ln\left(\frac{3+2\sqrt{3}}{3}\right).$$

$\therefore \quad k = \frac{1}{4} = 0.25$

29. (6) : $\lim_{x\to 0} \frac{2f(x)-3f(2x)+f(4x)}{x^2} \quad \left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x\to 0} \frac{2f'(x)-6f'(2x)+4f'(4x)}{2x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0} \frac{2f''(x)-12f''(2x)+16f''(4x)}{2}$$

$$= \frac{(2-12+16)2}{2} = 6$$

30. (0) : $\sin^{-1}x + \cos^{-1}y + \sec^{-1}z \le \frac{5\pi}{2}$

$$t^2 - \sqrt{2\pi}\,t + 3\pi = \left(t - \sqrt{\frac{\pi}{2}}\right)^2 + \frac{5\pi}{2} \ge \frac{5\pi}{2}$$

$\because$ Left side is decreasing and right side is increasing. So they will match only on their common point of incidence.

So solution exist if R.H.S. = L.H.S. $= \frac{5\pi}{2}$

i.e., $x = 1,\ y = -1,\ z = -1,\ t = \sqrt{\frac{\pi}{2}}.$

$\therefore \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z + \tan^{-1}\left(\sqrt{\frac{2}{\pi}}t\right)$

$$= \frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi}{4} = 0$$

◈◈

NOTIFICATION!

JEE Main 2021

The JEE (Main)-2021 is being conducted in multiple Sessions (February/March/April/May) for admissions in the next academic session.

New Scheme of Examination :

PAPER	SUBJECTS	Question Paper design		TIMING OF THE EXAMINATION(IST)	
		Section A	Section B	First Shift	Second Shift
B.E./ B.Tech. (Paper 1)	Mathematics	20*	10*	9:00 a.m. to 12:00 p.m.	3:00 p.m. to 6:00 p.m.
	Physics	20*	10*		
	Chemistry	20*	10*		
	Total	90			
B. Arch. (Paper 2A)	Mathematics – Part I	20*	10*	9:00 a.m. to 12:00 p.m.	3:00 p.m. to 6:00 p.m.
	Aptitude Test – Part II	50	-		
	Drawing Test – Part III	02	-		
	Total	82			
B. Planning (Paper 2B)	Mathematics – Part I	20*	10*	3:00 p.m. to 6:00 p.m.	
	Aptitude Test – Part II	50	-		
	Planning Based Questions - Part III	25	-		
	Total	105			

*Each Subject will have two sections. Section A will be of Multiple-Choice Questions (MCQs) and Section B will contain Questions whose answers are to be filled in as a numerical value. In Section B, candidates have to attempt any **five questions out of 10.** There will be **no negative marking for Section B.**

Mode of Examination : JEE (Main) 2021 Examination will be held in "Computer Based Test" (CBT) Mode only, except that the Drawing Test for B.Arch. will be held in "Pen & Paper" (offline) mode.

Medium of the Question Papers: Drawing from the National Education Policy (NEP), the JEE (Main) 2021 will be conducted in *Assamese, Bengali, Kannada, Malayalam, Marathi, Odia, Punjabi, Tamil, Telugu, Urdu in addition to Hindi, English and Gujarati.*

Dates of Examinations :

Session-1 (Februay-2021)	Session-2 (March-2021)	Session-3 (April-2021)	Session-4 (May-2021)
23,24,25 & 26 February 2021	15,16,17,& 18 March 2021	27,28,29 & 30 April 2021	24,25,26 27 & 28 May 2021

Important Instructions:

- Candidates can apply for JEE (Main) 2021 through **"Online" mode only between 16.12.2020 (Wednesday) to 16.01.2021 (Saturday)**
- The fees can be paid **online upto *17.01.2021 (Sunday).***
- Fees can be paid for all 4 Session(s) (February/March/April/May) at the same time. However, it can be taken forward to another Session. In case a candidate does not want to appear in the Session for which fee has already been paid, it will be refunded by NTA, provided the request is received during the application process of the Session from which the candidate wants to withdraw.
- A single Confirmation Page will be generated for all the session(s), the candidate applies for.
- Candidates has option to apply for one Session and more than one Session together and pay fee accordingly. Application window will be re-opened briefly immediately after the declaration of result of February/ March & April Session.
- If any candidate could not apply for session-1, he/she can apply for the next Session(s) once the portal opens, after declaration of result.
- **Only one application is to be submitted by a candidate for one session or more than one session or for all session(s) together. Multiple application forms submitted by a candidate for the same session will not be accepted at any cost.**
- For more information visit : **jeemain.nta.nic.in/www.nta.ac.in.**

JEE WORK OUTS

PAPER-I

SECTION-1

This section contains eight questions. The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. How many solutions are there for the equation, $|\cos x - \sin x| = 2\cos x$ in the interval $[0, 2\pi]$?

2. The number of solutions of the system $\log_2 x \log_y 2 + 1 = 0$ and $\sin x \cos y = 1 - \cos x \sin y$, which satisfy the condition $x + y < 8$ is

3. $\tan 100° + \tan 125° + \tan 100° \tan 125° =$

4. If $\cos 2\alpha = \dfrac{3\cos 2\beta - 1}{3 - \cos 2\beta}$, then $\left(\dfrac{\tan\alpha}{\tan\beta}\right)^2$ is equal to

5. If $A + B + C = 180°$,
$\dfrac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = k \sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$,
then the value of k is equal to

6. The expression $\tan 55° \cdot \tan 65° \cdot \tan 75°$ simplifies to $\cot x°$, where $x \in (0, 90)$ then x equals

7. The maximum value of $y = 2\sin^2 x - 3\sin x + 1 \ \forall\, x \in R$ is

8. Let x and y be positive real numbers and θ be an angle such that $\theta \neq \dfrac{n\pi}{2}$, $n \in I$. Suppose $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y}$ and $\dfrac{\cos^4\theta}{x^4} + \dfrac{\sin^4\theta}{y^4} = \dfrac{97\sin 2\theta}{x^3y + y^3x}$, then the value of $\left(\dfrac{x}{y} + \dfrac{y}{x}\right)$ is equal to

SECTION-2

This section contains ten questions. Each question has four options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct. For each question, darken the bubbles corresponding to all the correct options in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened. –2 In all other cases.

9. The vectors $\vec{a}, \vec{b}, \vec{c}$ are of the same length and the angle between any two of them is the same. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ then $\vec{c}$ is
   (a) $-\dfrac{1}{3}(\hat{i} - 4\hat{j} + \hat{k})$ (b) $\hat{i} + \hat{k}$
   (c) $\dfrac{1}{3}(-\hat{i} + 5\hat{j} + \hat{k})$ (d) $(\hat{i} + 2\hat{j} + 3\hat{k})$

10. The lines whose vector equations are
$\vec{r} = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + p\hat{j} + 5\hat{k})$ and
$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - p\hat{j} + p\hat{k})$
are perpendicular for all values of λ and μ if
   (a) $p = -6$ (b) $p = -1$
   (c) $p = 1$ (d) $p = 6$

11. The vectors $\vec{a} = x\hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} + y\hat{j} - z\hat{k}$ are collinear if
   (a) $x = 1, y = -2, z = -5$
   (b) $x = 1/2, y = -4, z = -10$
   (c) $x = -1/2, y = 4, z = 10$
   (d) $x = -1, y = 2, z = 5$

12. Vector equation of the line $6x - 2 = 3y + 1 = 2z - 2$ is
   (a) $\vec{r} = \hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(b) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right)$

(c) $\vec{r} = \frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(d) $\vec{r} = -2\hat{i} + \hat{j} - 2\hat{k} + \lambda(6\hat{i} + 3\hat{j} + 2\hat{k})$

where, λ being a parameter.

13. The equations of the line of shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ are

(a) $3(x - 21) = 3y + 92 = 3z - 32$

(b) $\frac{x-(62/3)}{1/3} = \frac{y+31}{1/3} = \frac{z-(31/3)}{1/3}$

(c) $\frac{x-21}{1/3} = \frac{y+(92/3)}{1/3} = \frac{z-(32/3)}{1/3}$

(d) $\frac{x-2}{1/3} = \frac{y+3}{1/3} = \frac{z-1}{1/3}$

14. If Δ denotes the area of the triangle ABC, then Δ is equal to

(a) $ab\sin C$ (b) $\frac{1}{2}a^2\frac{\sin B \sin C}{\sin(B+C)}$

(c) $\frac{1}{2}(a+b+c)r$ (d) $\frac{1}{2}(a+b-c)r$

15. $\sqrt{\cos 2x} + \sqrt{1+\sin 2x} = 2\sqrt{\sin x + \cos x}$ if

(a) $x = n\pi - \frac{\pi}{3}$ (b) $x = 2n\pi$

(c) $x = n\pi - \frac{\pi}{4}$ (d) $x = 2n\pi \pm \cos^{-1}\left(-\frac{1}{5}\right)$

16. If A and B are angles such that $A + B = \frac{\pi}{3}$ and $y = \tan A \tan B$, then y can be equal to

(a) 0 (b) 2 (c) 4 (d) 5

17. The equation $\sin^4 x + \cos^4 x = a$ has a solution for

(a) all values of a (b) $a = 1$

(c) $a = 1/2$ (d) $a = 3/4$

18. If $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left(\left(\frac{1}{3}\right)\tan\theta\right)$, then $\theta =$

(a) 0 (b) $\pi/4$

(c) $\tan^{-1}(-2)$ (d) none of these

SECTION - 3

This section contains TWO questions. Each question contains two columns, Column I and Column II. Column I has four entries (A), (B), (C) and (D). Column II has five entries (P), (Q), (R), (S) and (T). Match the entries in Column I with the entries in Column II. One or more entries in Column I may match with one or more entries in Column II. The ORS contains a 4 × 5 matrix whose layout will be similar to the one as shown.

For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).

(A) (P) (Q) (R) (S) (T)
(B) (P) (Q) (R) (S) (T)
(C) (P) (Q) (R) (S) (T)
(D) (P) (Q) (R) (S) (T)

Marking scheme : For each entry in Column I, +2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened; 0 If none of the bubbles is darkened; −1 In all other cases.

19. Match the following :

	Column I		Column II
A.	Maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) - 1$ is	P.	1
B.	$\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then $\tan t =$	Q.	6
C.	If $3\cos\theta - 4\sin\theta = 5$, then $3\sin\theta + 4\cos\theta =$	R.	2
D.	If $\sin A \sin B \sin C + \cos A \cos B = 1$, then $2\cos(A - B) =$	S.	0

20. Match the following:

	Column I		Column II
A.	In a ΔABC, $\cos A + \cos B + \cos C$ is equal to	P.	$\frac{a^2+b^2+c^2}{\Delta^2}$
B.	$\frac{r_1 - r}{a} + \frac{r_2 - r}{b} + \frac{r_3 - r}{c} =$	Q.	$1 + \frac{r}{R}$
C.	$\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} =$	R.	$\frac{r_1 + r_2 + r_3}{s}$
D.	$\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r^2}$	S.	$\frac{1}{r} - \frac{1}{2R}$

SECTION-1

This section contains eight questions. The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. If l is the shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$, then determine $[l]$ ($[x]$ denotes G.I.F)

2. Let $\vec{a}$ and $\vec{b}$ be unit vectors. If $\vec{c}$ is a vector such that $\vec{c}+(\vec{c}\times\vec{a})=\vec{b}$, then the maximum value of $|(\vec{a}\times\vec{b})\cdot\vec{c}|$ is $A\times10^{-1}$. Find A.

3. The straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular then the value of $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}$ is

4. If the distance of the point $B(\hat{i}+2\hat{j}+3\hat{k})$ from the line which is passing through $A(4\hat{i}+2\hat{j}+2\hat{k})$ and which is parallel to the vector $\vec{c}=2\hat{i}+3\hat{j}+6\hat{k}$ is λ then value of $\lambda^2 - 1$ is

5. Find the number of real solutions of the equation $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$.

6. Number of roots of the equation $\cos(\sin x)=\frac{1}{\sqrt{2}}$, $(0<x<\pi)$ is

7. Let x, y, z be real numbers such that $\cos x + \cos y + \cos z = 0$ and $\cos3x + \cos3y + \cos3z = 0$ then find the maximum value of $\cos2x\cos2y\cos2z$.

8. If $x + y = 2\pi/3$ and $\sin x/\sin y = 2$, then the number of values of $x \in [0, 4\pi]$ is

SECTION-2

This section contains eight questions. Each question has four options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct. For each question, darken the bubble(s) corresponding to all the correct options in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened, 0 if none of the bubbles is darkened, −2 In all other cases.

9. Y-coordinate of the point of intersection of the curves is, $y = \cos x$, $y = \sin3x$, $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ is
   (a) $\cos\left(\frac{\pi}{8}\right)$ (b) $\frac{1}{\sqrt{2}}$
   (c) $\sin\left(\frac{\pi}{8}\right)$ (d) $-\frac{1}{\sqrt{2}}$

10. The satisfactory value of x, where $x\in(-\pi,\pi)$ for the given equation, $(\sqrt{3}\sin x+\cos x)^{\sqrt{(\sqrt{3}\sin2x-\cos2x+2)}}=4$ is
    (a) $\frac{\pi}{3}$ (b) $-\frac{5\pi}{3}$ (c) $-\frac{2\pi}{5}$ (d) $-\frac{\pi}{3}$

11. If $\left(\cos^2x+\frac{1}{\cos^2x}\right)(1+\tan^2 2y)(3+\sin3z)=4$, then
    (a) x may be a multiple of π
    (b) x cannot be an odd multiple of π
    (c) y can be a multiple of $\pi/2$
    (d) z can be a multiple of $\pi/2$

12. The values of 'x' which satisfy the equation $2(\cos x + \cos2x) + \sin2x(1 + 2\cos x) = 2\sin x$, and $-\pi\le x\le\pi$ are
    (a) $\pi, -\pi$ (b) $-(\pi/3), (\pi/3)$
    (c) $-\pi/2$ (d) none of these

13. For $\theta\in[0, 2\pi]$, the values of 'θ' and 'y' satisfying the inequality, $2^{\frac{1}{\sin^2\theta}}\cdot\sqrt{y^2-2y+2}\le2$ are
    (a) $\left(\frac{\pi}{4},-1\right)$ (b) $\left(\frac{\pi}{2},1\right)$
    (c) $\left(\frac{3\pi}{2},1\right)$ (d) $\left(\frac{\pi}{4},1\right)$

14. The positive integral solutions (x, y) of $\tan^{-1}x+\cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right)=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ are
    (a) (1, 2) (b) (2, 1)
    (c) (2, 7) (d) (2, 2)

15. Solving the equation $\frac{\sqrt{3}}{2}\sin x-\cos x=\cos^2 x$, we get
    (a) $x = (2n + 1)\pi$ (b) $x=2n\pi+\frac{\pi}{3}$
    (c) $x=2n\pi-\frac{\pi}{3}$ (d) none of these

16. The roots of the equation $\left(\sin\frac{\pi}{6}\right)^{\cot 3x+2\cot x} = \sin^2(2\pi-x)-\cos(\pi-x)\times\sin\left(\frac{\pi}{2}+x\right)$ are

(a) $x = n\pi + \tan^{-1}\sqrt{\frac{7}{5}}$ (b) $x = n\pi - \tan^{-1}\sqrt{\frac{7}{5}}$

(c) $x = \tan^{-1}\sqrt{\frac{5}{7}}$ (d) none of these

SECTION-3

This section contains two paragraphs. Based on each paragraph, there will be two questions. Each question has four options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened ; 0 If none of the bubbles is darkened ; – 2 In all other cases.

Paragraph for Q. No. 17 & 18

Let '*k*' be the length of any edge of a regular tetrahedron. (A tetrahedron whose edges are all equal in length is called a regular tetrahedron). The angle between a line and a plane is equal to the complement of the angle between the line and the normal to the plane whereas the angle between two planes is equal to the angle between the normals. Let '*D*' be the origin be one of the vertex and *A*, *B* and *C* vertices with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively of the regular tetrahedron.

17. The angle between any edge and a face not containing the edge is

(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/4)$

(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\pi/3$

18. The angle between any two faces is

(a) $\cos^{-1}(1/\sqrt{3})$ (b) $\cos^{-1}(1/4)$

(c) $\pi/3$ (d) none of these

Paragraph for Q. No. 19 & 20

The circle touching *BC* and the two sides *AB* and *AC* produced of a triangle *ABC* is called the described circle opposite angle *A*. Its radius is denoted by r_1. Similarly, r_2 and r_3 denote the radii of the described circles opposite angles *B* and *C*, respectively. They are known as the ex-radii of triangle *ABC*. Here Δ is the area of a triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where $2s = a + b + c$. Also *R* is the radius of the circumcircle of a triangle *ABC* and *r* is the radius of the circle inscribed in triangle *ABC*.

19. Suppose in a triangle *ABC*, $a : b : c = 4 : 5 : 6$. The ratio of the radius of the circumcircle to that of the incircle is

(a) $\frac{1}{3}$ (b) $\frac{16}{7}$ (c) $\frac{3}{14}$ (d) $\frac{8}{19}$

20. Suppose in a triangle *ABC*, $8R^2 = a^2 + b^2 + c^2$, then the triangle *ABC* is

(a) right angled (b) isosceles

(c) equilateral (d) none of these

ANSWER KEY

PAPER-I

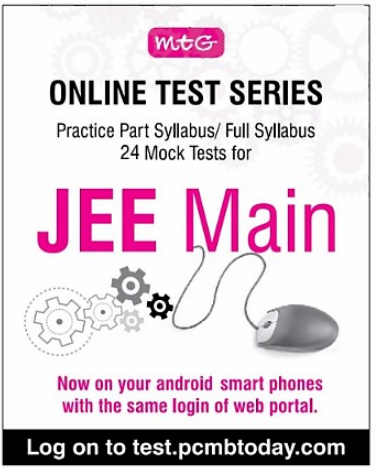
1.	(2)	2.	(2)	3.	(1)	4.	(2)	5.	(8)
6.	(5)	7.	(6)	8.	(4)	9.	(a, b)	10.	(b, d)
11.	(a, b, c, d)			12.	(c)	13.	(a, b, c)		
14.	(b, c)			15.	(b, c)	16.	(a, c, d)		
17.	(b, c, d)			18.	(a, b)				

19. (A) → (Q), (B) → (P), (C) → (S), (D) → (R)

20. (A) → (Q), (B) → (R), (C) → (S), (D) → (P)

PAPER-II

1.	(6)	2.	(5)	3.	(0)	4.	(9)	5.	(1)
6.	(2)	7.	(0)	8.	(4)	9.	(a, b, c)		
10.	(a, b)		11.	(a, c)		12.	(a, b, c)		
13.	(b, c)	14.	(a, c)	15.	(a, b)	16.	(a, b)	17.	(c)
18.	(d)	19.	(b)	20.	(a)				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The greatness of Srinivasa Ramanujan

His works were diverse, original and transcended time; and all this while he surmounted many odds

" In small proportions we just beauties see; And in short measures life may perfect be" - 'The Noble Nature,' Ben Jonson

In April 1984, when he read the letter thanking him for contributing to a bronze bust in memory of the mathematician Srinivasa Ramanujan, Takashi Ono was moved to tears. The letter was from Janaki Ammal, Ramanujan's wife. From first hearing about Ramanujan in Tokyo in 1955 from the mathematician Andre Weil and having Ramanujan as an inspiration and then to being one of the few mathematicians in the world to fulfil the wish of Janaki Ammal that a statue of her husband be made, life had come a full circle for Takashi.

There was one more connection that Takashi had with Ramanujan: Takashi's son Ken Ono made key contributions to 'mock theta functions' in early 2000s, a topic that Ramanujan had worked on in the last few months before his death in April 1920. In the mathematical community, it was no surprise that Ramanujan's work was being expanded actively eight decades after his demise: many of Ramanujan's findings anticipated research areas by many years.

Research at a feverish pitch

Ramanujan, born on December 22, 1887 was an autodidact who specialised in pure mathematics. While he excelled in mathematics, he neglected other subjects and could not complete his pre-university course. By 1908 he gave up studies, but not his research in mathematics. He struggled in poverty until in 1910, a benefactor, Ramachandra Rao, district collector of Nellore, provided him monthly allowance from his own pocket so that Ramanujan could pursue research. This would continue for a couple of years until Ramanujan managed to become a clerk at Madras Port Trust. He initiated contact with the British mathematician G.H. Hardy under whose insistence Ramanujan travelled to England in early 1914. His partnering with Hardy was productive: Ramanujan published more than 20 research papers between 1914 and 1919. During his stay, he was awarded a doctorate and made Fellow of Royal Society. When he returned to India in 1919, he was "...with a scientific standing and reputation such as no Indian has enjoyed before". Unfortunately he lived only a year after his return succumbing to illness which was diagnosed then as tuberculosis but now revised as hepatic amoebiasis. However, in that one year, he continued his research at a feverish pitch.

Until he left for England in 1914, Ramanujan recorded his mathematical results, mostly equations, in his notebooks. There were three such notebooks (preserved now). One more was added when Ramanujan returned to India. Together there were about 4,000 results. The results were the culmination of research backed by deep intuition and insights. However, Ramanujan did not record proofs of his results: that work would be taken up by future generations of mathematicians.

Ramanujan's work was in number theory, infinite series, analysis (theoretical underpinnings of calculus) and a few other areas in pure mathematics. Specifically, as G.H. Hardy wrote, these subjects were "...the applications of elliptic functions to the theory of numbers, the theory of continued fractions and... the theory of partitions".

A few significant contributions were multiple formulae to calculate pi with great accuracy to billions of digits (22/7 is only an approximation to pi), partition functions (a partition is a way to represent a positive integer — for example, 1+1+1+1 is a partition of 4, 1+3 is another partition of 4, and so on), modular forms and hypergeometric series (the terms in every consecutive pair in the series form rational functions).

The importance of many of his works became apparent much later. One such was 'Ramanujan conjecture' which he published in 1916 and was proved in 1973 by Pierre Deligne. The conjecture inspired the development of theory of Galois representation that was employed in Andrew Wiles' proof of Fermat's last theorem published in 1995. In recent years, Ramanujan's works and their extensions have found applications in signal processing to identify periodic information, akin to Fourier analysis. Mock theta functions have found applications in the study of black holes in astrophysics.

But to look for applications of his works is exactly how not to appreciate Ramanujan. As G.N. Watson, a contemporary mathematician of Ramanujan in Cambridge said, "The study of Ramanujan's works gives me a thrill which is indistinguishable from the thrill which I feel... when I see before me the... beauty of the four statues... which Michelangelo has set over the tombs of Guiliano de Medici and Lorenzo de Medici". For Ramanujan, mathematics was art.

Robert Kanigel, the celebrated biographer of Ramanujan, noted: "People will try to explain it in an easy way but I think they are unjustified in doing it. I think some people really are a few steps beyond where the rest of us live. We are forced to view those intellects, those artistic sensibilities, as a little bit mysterious or a little beyond what is the common realm."

For Ramanujan, his mathematics was an end in itself. In this he thought like G.H. Hardy who claimed in A Mathematician's Apology, "A mathematician, like a painter or a poet, is a maker of patterns... The 'seriousness' of a mathematical theorem lies, not in its practical consequences, which are usually negligible, but in the significance of the mathematical ideas which it connects. We may say, roughly, that a mathematical idea is 'significant' if it can be connected, in a natural and illuminating way, with a large complex of other mathematical ideas."

How did Ramanujan develop his mathematical abilities? Here again we have to look to G.H. Hardy: "...with his memory, his patience and his power of calculation, he combined a power of generalisation, a feeling for form, a capacity for rapid modification of his hypothesis, that were often really startling, and made him, in his own peculiar field, without a rival in his day".

Power of intuition and insight

Let's remember that Ramanujan was always precocious in his mathematical talent. And by virtue of working alone on problems and theorems that were advanced for his age during adolescence, day after day, hour after hour, he developed an incredible power of intuition and insight. The problems he worked on were from a book by one Carr. This book was "a spark which'ignited the flame... [but] as the depth of Ramanujan's discoveries deepened, Carr's influence certainly waned," the mathematician Bruce Berndt said.

Ramanujan is remembered in many ways. His birthday is celebrated as National Mathematics Day in India. The Ramanujan Journal publishes advancements in the areas that Ramanujan contributed to. His home in Kumbakonam in Tamil Nadu has been converted to a museum by SASTRA University.

Ramanujan's greatness is not that he was a poor man who travelled to England and did research. It is that he was prolific and that his works were diverse, original and transcended time; and all this while he surmounted many odds coming from indigence. Ramanujan's works, especially to mathematicians, are of enduring elegance. The mathematician and physicist Freeman Dyson said, "Whenever I am angry or depressed, I pull down the Collected papers [of Ramanujan] from the shelf and take a quiet stroll in Ramanujan's garden".

Courtesy : Varahasimhan

OLYMPIAD CORNER

1. A positive integer is called monotonic if its digits in base 10, read from left to right, are in non-decreasing order. Prove that for each $n \in N$, there exists an n-digit monotonic number which is a perfect square.

2. Let P be the intersection point of the diagonals $\overline{AC}$ and BD of the convex quadrilateral $ABCD$ in which $AB = AC = BD$. Let O and I be the circumcenter and incenter, respectively, of triangle ABP. Prove that if $O \neq I$, then lines OI and CD are perpendicular.

3. The incircle of the isosceles triangle ABC touches the legs $\overline{AC}$ and $\overline{BC}$ at points M and N, respectively. A line t is drawn tangent to minor arc MN, intersecting $\overline{NC}$ and $\overline{MC}$ at points P and Q, respectively. Let T be the intersection point of lines AP and BQ.

(a) Prove that T lies on $\overline{MN}$.

(b) Prove that the sum of the areas of triangles ATQ and BTP is smallest when t is parallel to line AB.

4. The feet of the angle bisectors of triangle ABC are X, Y, and Z. The circumcircle of triangle XYZ cuts off there segments from lines AB, BC and CA. Prove that two of these segments' lengths add up to the third segment's length.

5. An exam paper consists of 5 multiple-choice questions, each with 4 different choices; 2000 students take the test, and each student chooses exactly one answer per question. Find the smallest value of n for which it is possible for the students' answer sheets to have the following property: among any n of the students' answer sheets, there exist 4 of them among which any two have at most 3 common answers.

6. Let m, n be positive integers such that $m \leq \frac{n^2}{4}$ and every prime divisor of m is less than or equal to n. Show that m divides $n!$.

7. Let p be a prime number such that $p \equiv 1 \pmod 4$. Evaluate

$$\sum_{k=1}^{p-1}\left(\left\lfloor \frac{2k^2}{p} \right\rfloor - 2\left\lfloor \frac{k^2}{p} \right\rfloor\right)$$

8. The real numbers a, b, c, x, y, z satisfy $a \geq b \geq c > 0$ and $x \geq y \geq z > 0$. Prove that

$$\frac{a^2x^2}{(by+cz)(bz+cy)} + \frac{b^2y^2}{(cz+ax)(cx+az)} + \frac{c^2z^2}{(ax+by)(ay+bx)}$$

is at least $\frac{3}{4}$.

9. If $g(x)$ and $h(x)$ are polynomials with real coefficients and $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, show that $g(x)$ and $h(x)$ are both divisible by $x - 1$.

10. In a triangle ABC, and A is twice of angle B. Show that $a^2 = b(b + c)$.

11. M is an interior point of a triangle ABC. Bisectors of interior angles BMC, CMA, AMB intersect BC, CA, AB respectively at X, Y, Z. Show that AX, BY, CZ are concurrent. If P is the point of concurrence and $\frac{PA}{PX}\cdot\frac{PB}{PY}\cdot\frac{PC}{PZ} = 8$, then show that M is the circumcenter of ΔABC.

12. If $A_1, A_2, \ldots, A_n$ is an n - sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, find n, the number of sides of the polygon.

SOLUTIONS

1. Any 1-digit perfect square (namely, 1, 4, or 9) is monotonic, proving the claim for $n = 1$. We now assume $n > 1$. If n is odd, write $n = 2k - 1$ for an integer $k \leq 2$, and let

$x_k = \dfrac{\left(10^k+2\right)}{6} = 1\underbrace{66\ldots67}_{k-2}$.

Then, $x_k^2 = \dfrac{10^{2k}+4\cdot10^k+4}{36} = \dfrac{10^{2k}}{36}+\dfrac{10^k}{9}+\dfrac{1}{9}$. (*)

Observe that $\dfrac{10^{2k}}{36} = 10^{2k-2}\left(\dfrac{72}{36}+\dfrac{28}{36}\right)$

$= 2\cdot10^{2k-2}+10^{2k-2}\cdot\dfrac{7}{9} = 2\underbrace{77\ldots7}_{2k-2}+\dfrac{7}{9}$.

Thus, the right-hand side of (*) equals

$\left(2\underbrace{77\ldots7}_{2k-2}+\dfrac{7}{9}\right)+\left(\underbrace{11\ldots1}_{k}+\dfrac{1}{9}\right)+\dfrac{1}{9} = 2\underbrace{77\ldots7}_{k-2}\underbrace{88\ldots89}_{k-1}$,

an n-digit monotonic perfect square. If n is even, write $n = 2k$ for an integer $k \geq 1$, and let

$y_k = \dfrac{10^k+2}{3} = \underbrace{33\ldots34}_{k-1}$

Then, $y_k^2 = \dfrac{\left(10^{2k}+4\cdot10^k+4\right)}{9} = \dfrac{10^{2k}}{9}+4\cdot\dfrac{10^k}{9}+\dfrac{4}{9}$

$= \left(\underbrace{11\ldots1}_{2k}+\dfrac{1}{9}\right)+\left(\underbrace{44\ldots4}_{k}+\dfrac{4}{9}\right)+\dfrac{4}{9} = \underbrace{11\ldots1}_{k}\underbrace{55\ldots56}_{k-1}$

an n-digit monotonic perfect square. This completes the proof.

2. We first prove a fact that is very helpful in proving that two segments are perpendicular: Given two segments $\overline{XY}$ and $\overline{UV}$, $\overline{XY} \perp \overline{UV}$, if and only if $UX^2 - XV^2 = UY^2 - YV^2$. To prove this result, let X' and Y' be the feet of the perpendiculars of X and Y, respectively, to line UV. Then $\overline{XY} \perp \overline{UV}$ if and only if $X' = Y'$, that is, if and only if

$UX' - X'V = UY' - Y'V$ (*)

where all distances are directed distances. Because $UX' + X'V = UV = UY' + Y'V$, equation (*) holds if and only if $UX'^2 - X'V' = UY'^2 - Y'V^2$, or equivalently $UX^2 - XV^2 = UY^2 - YV^2$. Thus, it suffices to show that $DO^2 - CO^2 = DI^2 - CI^2$. Let $AB = AC = BD = p$, $PC = a$ and $PD = b$. Then, $AP = p - a$ and $BP = p - b$. Let R be the circumradius of triangle ABP. *By* the Power of a Point Theorem $pb = DP\cdot DB = DO^2 - R^2$. Likewise, $pa = CO^2 - R^2$. Hence, $DO^2 - CO^2 = p(b - a)$. Because triangle ABD is isosceles with $BA = BD$, and I lies on the bisector of angle ABD, $ID = IA$. Likewise, $IB = IC$. Let T be the point of tangency of the incircle of triangle ABC to side $\overline{AB}$. Then $AT = \dfrac{(AB+AP-BP)}{2} = \dfrac{(p+b-a)}{2}$ and $BT = \dfrac{(p+a-b)}{2}$.

Because $IT \perp AB$, $AI^2 - BI^2 = AT^2 - BT^2$. Putting the above arguments together, we find that $DI^2 = CI^2$

$= AI^2 - BI^2 = AT^2 - BT^2 = (AT + BT)(AT - BT)$

$= p(b - a) = DO^2 - CO^2$, as desired.

3. (a) The degenerate hexagon $AMQPNB$ is circumscribed about the incircle of triangle ABC. By Brianchon's Theorem, its diagonals $\overline{AP}$, $\overline{MN}$ and $\overline{QB}$ concurrent. Therefore, T lies on $\overline{MN}$.

One can also use a more elementary approach. Let R and S be the points of tangency of the incircle with sides $\overline{AB}$ and $\overline{PQ}$, respectively. Let $\overline{BQ}$ intersect $\overline{MN}$ and $\overline{SR}$ at T_1 and T_2, respectively. Because $\angle QMN = \angle PNM = \dfrac{\overset{\frown}{MN}}{2}$, we have $\sin\angle QMN = \sin\angle PNM = \sin\angle BNM$. Applying the Law of Sines to triangles MQT_1 and NBT_1 yields

$\dfrac{QT_1}{QM} = \dfrac{\sin\angle QMN}{\sin\angle QT_1M} = \dfrac{\sin\angle BNM}{\sin\angle BT_1N} = \dfrac{BT_1}{BN}$ or $\dfrac{QT_1}{BT_1} = \dfrac{MQ}{BN}$

Likewise, $\dfrac{QT_2}{BT_2} = \dfrac{SQ}{BR}$

By equal tangents, $BN = BR$ and $QM = QS$. Hence $\dfrac{QT_1}{BT_1} = \dfrac{QT_2}{BT_2}$.

Because T_1 and T_2 both lie on $\overline{BQ}$, we must have $T_1 = T_2$. Hence, $\overline{BQ}$, $\overline{MN}$, $\overline{SR}$ are concurrent. In exactly the same manner, we can prove that $\overline{AP}$, $\overline{MN}$, $\overline{SR}$ are concurrent. It follows that T lies on $\overline{MN}$.

(b) Let $\alpha = \angle CAB = \angle CBA$ and $\beta = \angle ACB$. Let $f = [AQT]+[BPT] = [ABQ]+[ABP]-2[ABT]$. Because triangle ABC is isosceles, $\overline{MN} \| \overline{AB}$, implying that $[ABT]$ is constant. Hence, minimizing f is equivalent to minimizing $g = [ABQ] + [ABP]$. Note that $2g = AB(AQ + PB)\sin\alpha = AB(AB + PQ)\sin\alpha$ where $AQ + PB = AB + QP$ because quadrilateral $ABCD$, has an inscribed circle. Thus, it suffices to minimize PQ.

Let I be the incenter of triangle ABC, so that I is the excenter of triangle CPQ opposite C. Hence, $PC + CQ + QP = 2CM$ is constant. Let $\angle CPQ = p$ and $\angle CPQ = q$. Then, $p + q = \pi - \beta$ is constant as well. Applying the Law of Sines to triangle CPQ yields

$$\frac{CM}{PQ} = 1+\frac{CP}{PQ}+\frac{CQ}{PQ} = 1+\frac{\sin p+\sin q}{\sin\beta}$$

$$= 1+\frac{2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}}{\sin\beta}$$

Hence, it suffices to maximize $\cos\dfrac{p-q}{2}$. It follows that $[ATQ] + [BTP]$ is minimized when $p = q$, that is, when $\overline{PQ} \| \overline{AB}$.

4. We use signed distances throughout the problem, where $a = BC$, $b = CA$, and $c = AB$ are positive. Also, let the specified circle intersect BC at X and P, CA at Y and Q, and AB at Z and R and let $x = PX$, $y = QY$, $z = RZ$. By the angle bisector theorem, $YA = \dfrac{bc}{(c+a)}$ and $AZ = \dfrac{bc}{(a+b)}$. Hence,

$$QA=\frac{bc}{(c+a)+y} \text{ and } AR=\frac{bc}{(a+b)-z}.$$

Applying the Power of a Point Theorem to point A and the specified circle gives

$$\frac{bc}{c+a}\left(\frac{bc}{c+a}+y\right)=\frac{bc}{a+b}\left(\frac{bc}{a+b}-z\right).$$

After multiplying through out by $\frac{a}{bc}$ and rearranging, we obtain

$$\frac{a}{c+a}y+\frac{a}{a+d}z=\frac{abc}{(a+b)^2}-\frac{abc}{(c+a)^2}.$$

Similarly, we have

$$\frac{b}{a+b}z+\frac{b}{b+c}x=\frac{abc}{(b+c)^2}-\frac{abc}{(a+b)^2}.$$

$$\frac{c}{b+c}x+\frac{c}{c+a}y=\frac{abc}{(c+a)^2}-\frac{abc}{(b+c)^2}.$$

When we add these three equations, we get $x + y + z = 0$. It follows that two of the quantities x, y, z are of the same sign and that the third is of the other sign. Hence, the sum of the absolute values of the former two quantities equals the absolute value of the latter, which is what we wished to prove.

5. First we prove that $n \geq 25$. Let 1, 2, 3, 4 denote the four different choices of each problem. Represent each student's answer sheet by an ordered 5-tuple $(a_1, a_2, a_3, a_4, a_5)$, $a_i \in \{1, 2, 3, 4\}$, where the student's answer to problem i is a_i. We say that two answer sheets are of the same type if their corresponding 5-tuples belong to a set of the form

$$\{(k, a_2, a_3, a_4, a_5) | k \in \{1, 2, 3, 4\}\},$$

where $a_2, a_3, a_4, a_5 \in \{1, 2, 3, 4\}$. Since there are 256 such sets and $2000 = 256 \times 7 + 208$, at least eight answer sheets are of the same type by the pigeonhole Principle. Among the 1992 remaining answer sheets, again some eight are of the same type. Finally, among the 1984 remaining answer sheets, another eight are of the same type. Consider the set A of these 24 answer sheets. Given any four answer sheets in A, two of them must be of the same type, that is, their solutions for the last 4 problems are identical. This violates the assumption that there are 4 answer sheets in A, among which any two have at most 3 common answers. Hence, $n \geq 25$.

Now we show that $n = 25$ is indeed attainable. Define the set $S = \{(a_1, a_2, a_3, a_4, a_5) | \sum_{i=1}^{5} a_i \equiv 0 \pmod 4, a_i \in \{1, 2, 3, 4\}\}$. Then $|S| = 4^4 = 256$, and any two answer sheets have at most 3 common answers if their corresponding 5-tuples are distinct elements of S. Pick any 250 elements of S and assume that exactly eight students turn in answer sheets that correspond to each of these 250 5-tuples. Among any $25 > 3.8$ answer sheets, there are four whose corresponding 5-tuples are distinct elements in S, and they satisfy the given conditions of the problem.

Therefore, the answer is $n = 25$.

6. It suffices to show that $p^k | n!$ for all primes p and integers $k \geq 1$ such that $p^k | m$. If $k = 1$, we are done because we are given that $p \leq n$ and hence $p | n!$. Otherwise, because $m \leq \frac{n^2}{4}$, we have $p^k \leq \frac{n^2}{4}$. Thus, $n \geq 2\sqrt{p^k}$. If $n \geq kp$, then at least k of the numbers 1, 2, ..., n are multiples of p, implying that $p^k | n!$. Hence, it suffices to show that $2\frac{\sqrt{p^k}}{p} \geq k$ or equivalently that

$$p^{\frac{(k-2)}{2}} \geq \frac{k}{2}, \qquad (*)$$

because this implies $n \geq 2\sqrt{p^k} \geq kp$.

Indeed, we can do so for most values p and k. If $k = 2$, then (*) reads $1 \geq 1$. If $k \geq 4$, then Bernoulli's inequality implies that $p^{\frac{(k-2)}{2}} = (1+(p-1))^{\frac{(k-2)}{2}} \geq 1+\frac{k-2}{2}(p-1) \geq \frac{k}{2}$, as desired. Finally, if $k = 3$, then (*) is clearly true unless $p = 2$.

If indeed $k = 3$ and $p = 2$, then $m \geq 8$, $n \geq 6$ and $n!$ is certainly divisible by 8. Thus, we have shown that $p^k | n!$ in all cases, completing the proof.

7. For all real x, let $\{x\} = x - [x] \in [0, 1)$. Writing

$$\left\lfloor \frac{2k^2}{p} \right\rfloor = \frac{2k^2}{p} - \left\{\frac{2k^2}{p}\right\} \text{ and } \left\lfloor \frac{k^2}{p} \right\rfloor = \frac{k^2}{p} - \left\{\frac{k^2}{p}\right\},$$

we find that $\left\lfloor \frac{2k^2}{p} \right\rfloor - 2\left\lfloor \frac{k^2}{p} \right\rfloor = 2\left\{\frac{k^2}{p}\right\} - \left\{\frac{2k^2}{p}\right\}$

When $\{x\} < \frac{1}{2}$, $2\{x\} - 2\{x\} = 2\{x\} - 2\{x\} = 0$.

When $\{x\} \geq \frac{1}{2}$, $2\{x\} - \{2x\} = 2\{x\} - (2\{x\} - 1) = 1$. Therefore, the desired sum equals the number $\propto$ of k in $[1, p - 1]$ such that $\left\{\frac{k^2}{p}\right\} \geq \frac{1}{2}$, or equivalently, the number of nonzero residues k modulo p such that k^2 is congruent to some number in $\left[\frac{(p+1)}{2}, p-1\right]$ modulo p.

Because p is a prime congruent to 1 modulo 4, it is well known that $-1 \equiv d^2 \pmod p$ for some integer d. Partition the nonzero residues modulo p into $\frac{p-1}{2}$ pairs of the form $\{a, da\}$ so that $a^2 \equiv -(da)^2 \pmod p$ in each pair. Thus, exactly one residue in each pair has a square congruent to some number in $\left[\frac{p+1}{2}, p-1\right]$, for a total of $\frac{p-1}{2}$ such residues. It follows that the given sum equals $\frac{p-1}{2}$.

8. Denote the left-hand side of the given inequality by S. Because $a \geq b \geq c$ and $x \geq y \geq z$ by the rearrangement inequality we have $bz + cy \leq by + cz$ so
$(by + cz)(bz + cy) \leq (by + cz)^2 \leq 2((by)^2 + (cz)^2)$.
Setting $\alpha = (ax)^2$, $\beta = (by)^2$, $\gamma = (cz)^2$, we obtain

$$\frac{a^2x^2}{(by+cz)(bz+cy)} \geq \frac{a^2x^2}{2\left((by)^2+(cz)^2\right)} = \frac{\alpha}{2(\beta+\gamma)}.$$

Adding to this the two analogous inequalities for the other summands, we find that

$$S \geq \frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right).$$

By the Cauchy-Schwarz inequality,

$$\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right)(\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta))$$

is at least $(\alpha + \beta + \gamma)^2$, which in turn equals

$$\frac{1}{2}\left((\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\right)+3(\alpha\beta+\beta\gamma+\gamma\alpha)$$

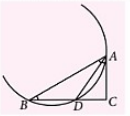
$$\geq \frac{3}{2}(2\alpha\beta+2\beta\gamma+2\gamma\alpha).$$

Therefore,

$$S \geq \frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right) \geq \frac{1}{2}\frac{(\alpha+\beta+\gamma)^2}{(2\alpha\beta+2\beta\gamma+2\gamma\alpha)} \geq \frac{3}{4},$$ as desired.

9. **1st solution:** If $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, we can write $f(x) = q(x)(x^2 + x + 1)$. Let ω be a non - real cube root of unity. Then $f(\omega) = q(\omega)(\omega^2 + \omega + 1) = 0$ and $f(\omega^2) = q(\omega^2)(\omega + \omega^2 + 1) = 0$. Therefore $g(1) + \omega h(1) = 0$ and $q(1) + \omega^2 h(1) = 0$. So, it follows that $g(1) = h(1) = 0$; therefore both $g(x)$ and $h(x)$ are divisible by $x - 1$.
2nd solution: we can write $g(x) = (x - 1)g'(x) + g(1)$.
Then, $g(x^3) = (x^3 - 1)\, g'(x^3) + g(1)$. Similarly, we can write $h(x^3) = (x^3 - 1)\, h'(x^3) + h(1)$. Therefore, $f(x) = (x^3 - 1)\, g'(x^3) + xh'(x^3)) + g(1) + xh(1)$.
Since $x^2 + x + 1$ divides both $f(x)$ and $x^3 - 1$, it also divides $g(1) + xh(1)$. Therefore $g(1) = h(1) = 0$.

10. **1st solution:** Let AD be the internal bisector of $\angle A$, meeting BC at D. Then $\frac{BD}{DC} = \frac{AB}{AC}$; adding 1 to both sides we get $\frac{BD}{DC} = \frac{AB+AC}{AC}$ ...(i)

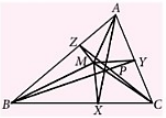


Since, $\angle CAD = \angle ABD$, AC is tangent to the circumcircle of ΔABD. Therefore, $BC.DC = AC^2$. ...(ii)
Now (i) × (ii) $\Rightarrow BC^2 = AC(AB + AC)$.
2nd solution: $A = 2B \Rightarrow C = \pi - 3B$ $\sin C = \sin 3B$. Therefore, $\sin^2 A = \sin^2 2B = 2\sin B\cos B \sin 2B$
$= \sin B(\sin B + \sin 3B) = \sin B(\sin B + \sin C)$
$\Rightarrow a^2 = b(b + c)$.

11. Since MX is the internal bisector of $\angle BMC$, $\frac{BX}{XC} = \frac{MB}{MC}$; similarly, $\frac{CY}{YA} = \frac{MC}{MA}$ and $\frac{AZ}{ZB} = \frac{MA}{MB}$.

Therefore, $\frac{BX}{XC}.\frac{CY}{YA}.\frac{AZ}{ZB} = 1$.

Hence by Ceva's theorem, AX, BY, CZ are concurrent.
Let $\Delta BPC = t_1$, $\Delta APC = t_2$ and $\Delta APB = t_3$.



Then,

$$\frac{PA}{PX} = \frac{\Delta APB}{\Delta BPX} = \frac{\Delta APC}{\Delta PXC} = \frac{\Delta APB + \Delta APC}{\Delta BPX + \Delta PXC} = \frac{t_2 + t_3}{t_1}.$$

Similarly, $\frac{PB}{PY} = \frac{t_3 + t_1}{t_2}$ and $\frac{PC}{PZ} = \frac{t_1 + t_2}{t_3}$.

Now,

$$8 = \frac{(t_2+t_3)(t_3+t_1)(t_1+t_2)}{t_1t_2t_3} \geq \frac{2\sqrt{t_2t_3}.2\sqrt{t_3t_1}.2\sqrt{t_1t_2}}{t_1t_2t_3}$$

(Equality holds only when $t_1 = t_2 = t_3 = 8$)
Thus, $t_1 = t_2 = t_3$. Now, $\frac{BX}{XC} = \frac{\Delta ABX}{\Delta ACX}$

$$= \frac{\Delta PBX}{\Delta PCX} = \frac{\Delta ABX - \Delta PBX}{\Delta ACX - \Delta PCX} = \frac{t_3}{t_2} = 1$$

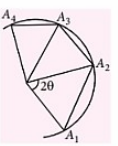
i.e., $BX = XC$. Since MB is the bisector of $\angle BMC$, it follows that $MB = MC$. Similarly $MC = MA$. Therefore, M is the circumcenter. (Note that P is the centroid).

12. We can assume that the radius of the circumcircle of the polygon is 1. Let 2θ be the angle subtended by any side of the polygon at the center of the circumcircle. We know, when a segment of the circle subtends an angle ϕ at the center, its length is $2\sin\frac{\phi}{2}$. Therefore, $A_1A_2 = 2\sin\theta$, $A_1A_3 = 2\sin 2\theta$ and $A_1A_4 = 2\sin 3\theta$. Now by hypothesis,

$$\frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{\sin 3\theta + \sin 2\theta}{\sin 2\theta \sin 3\theta}$$

$$\Rightarrow \sin 3\theta + \sin 2\theta = 2\sin 3\theta\cos\theta$$
$$= \sin 4\theta + \sin 2\theta$$
$$\Rightarrow \sin 3\theta + \sin 4\theta$$



Since $3\theta, 4\theta \in [0, \pi]$, we get $3\theta + 4\theta = \pi = n\theta$
$\Rightarrow n = 7$.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Evaluate the following indefinite integral:

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$ *(Priya Malhotra, Gujarat)*

Ans. Let $I = \int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)^5\sqrt{(x+1)^2-1}}$

$$= \int \frac{dx}{(x+1)^6\sqrt{1-\frac{1}{(x+1)^2}}} = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} \quad \left[\text{Putting } \frac{1}{x+1} = t\right]$$

Here, t^4 is a polynomial of degree 4.

$$\therefore \quad I = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} = (At^3+Bt^2+Ct+D)\sqrt{1-t^2} + \lambda \int \frac{dt}{\sqrt{1-t^2}} \quad \ldots(i)$$

Differentiating both sides and multiplying by $\sqrt{1-t^2}$, we get

$-t^4 = (3At^2+2Bt+C)(1-t^2) - t(At^3+Bt^2+Ct+D) + \lambda$

Comparing the coefficients, we have

$-4A = -1, 3B = 0, 3A - 2C = 0, 2B - D = 0$ and $C + \lambda = 0$

$\Rightarrow A = 1/4, B = 0, C = 3/8, D = 0$ and $\lambda = -3/8$

From (i) $I = \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} - \frac{3}{8}\int \frac{dt}{\sqrt{1-t^2}}$

$$= \frac{1}{8}(2t^3+3t)\sqrt{1-t^2} - \frac{3}{8}\sin^{-1}t + C, \text{ where } t = \frac{1}{x+1}.$$

2. Find the range of the function:

$$y = \frac{1}{2+\sin 3x+\cos 3x}.$$ *(Sidhartha Sharma, Bihar)*

Ans. We have,

$$y = \frac{1}{2+\sin 3x+\cos 3x} = \frac{1}{2+\sqrt{2}\sin(3x+\pi/4)}$$

The value of $\sqrt{2}\sin(3x+\pi/4)$ lies between $-\sqrt{2}$ and $\sqrt{2}$ for all $x \in R$; the denominator never becomes zero. Hence, the domain is $x \in R$.

Now, $\sqrt{2}\sin(3x+\pi/4) = \frac{1}{y} - 2$

For values of x belonging to the domain ($x \in R$), we have

$-\sqrt{2} \le \sqrt{2}\sin(3x+\pi/4) \le \sqrt{2}$

i.e., $-\sqrt{2} \le \frac{1}{y} - 2 \le \sqrt{2} \Rightarrow 2-\sqrt{2} \le \frac{1}{y} \le 2+\sqrt{2}$

$$\Rightarrow \quad \frac{1}{2+\sqrt{2}} \le y \le \frac{1}{2-\sqrt{2}}$$

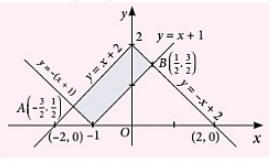
Hence, the range is $\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$.

3. Find the area enclosed by the curves $y = |x+1|$ and $y = 2 - |x|$. *(Mukesh Gupta, Delhi)*

Ans. We have, $y = |x+1| = \begin{cases} -(x+1), & x \le -1 \\ x+1, & x > -1 \end{cases}$

and $y = -|x| + 2 = \begin{cases} x+2, & x \le 0 \\ -x+2, & x > 0 \end{cases}$

Point of intersection of $y = x + 2$ and $y = -(x + 1)$ is A *i.e.*, $\left(\frac{-3}{2}, \frac{1}{2}\right)$ and point of interaction of $y = -x + 2$ and $y = x + 1$ is B *i.e.*, $\left(\frac{1}{2}, \frac{3}{2}\right)$.



Required area = area of shaded region

$$= \int_{-3/2}^{-1}[(x+2)-(-x-1)]\,dx + \int_{-1}^{0}[(x+2)-(x+1)]\,dx + \int_{0}^{1/2}[(-x+2)-(x+1)]\,dx$$

$$= \int_{-3/2}^{-1}(2x+3)\,dx + \int_{-1}^{0}1\,dx + \int_{0}^{1/2}(-2x+1)\,dx$$

$$= \left[x^2+3x\right]_{-3/2}^{-1} + [x]_{-1}^{0} + \left[-x^2+x\right]_{0}^{1/2}$$

$$= (1-3) - \left(\frac{9}{4}-\frac{9}{2}\right) + 1 + \left(\frac{-1}{4}+\frac{1}{2}\right) = \frac{3}{2} \text{ sq.units.}$$

Monthly Test Drive-8 CLASS XI ANSWER KEY

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(b)
6.	(c)	**7.**	(b,c)	**8.**	(a,b,d)	**9.**	(a,b)	**10.**	(b,c,d)
11.	(a,c)	**12.**	(b,d)	**13.**	(a,b)	**14.**	(a)	**15.**	(b)
16.	(b)	**17.**	(8)	**18.**	(6069)	**19.**	(190)	**20.**	(5)

CONCEPT BOOSTER

This column is aimed for students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR

Complex Numbers

COMPLEX NUMBERS

- Let us denote $\sqrt{-1}$ by the symbol i. Then $\pm i$ are the solutions of the equation $x^2 + 1 = 0$. Note that $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$ etc. A number of the form $a + ib$, where a and b are real numbers, is defined to be a complex number. For example $3 + 2i$, $-1 + \sqrt{7}\,i$, $-5i$, $2 = 2 + 0i$ are complex numbers. Note that every real number is a complex number.
- For the complex number $z = a + ib$, a is called the real part, denoted by $\text{Re}(z)$ and b is called the imaginary part denoted by $\text{Im}(z)$. A complex number z is said to be real if $\text{Im}(z) = 0$ and is said to be purely imaginary if $\text{Re}(z) = 0$. Thus the complex number $0 = 0 + 0i$ is both real and purely imaginary. $a + ib$ is called an imaginary number if $b \neq 0$.
- Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are equal if and only if $a = c$ and $b = d$. However there is no order relation in the set of complex numbers *i.e.*, the expressions of the form $a + ib < c + id$ and $a + ib > c + id$ are meaningless unless $b = d = 0$.
- **Algebra of Complex Numbers :** If $a, b, c, d, \lambda \in R$, then
  - **Addition :** $(a + ib) + (c + id) = a + c + i(b + d)$
  - **Subtraction :** $(a + ib) - (c + id) = a - c + i(b - d)$
  - **Multiplication :** $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$
  - **Division :** For $c + id \neq 0$,

$$\frac{a+ib}{c+id} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

  - **Multiplication by a real number :** $\lambda(a + ib) = \lambda a + i(\lambda b)$

CONJUGATE OF A COMPLEX NUMBER

- Let $z = a + ib$ be a complex number. The conjugate of z, denoted by $\bar{z}$, is the complex number $a - ib$, *i.e.*, $\bar{z} = a - ib$. Thus, $a = \text{Re}(z) = \dfrac{z+\bar{z}}{2}$ and $b = \text{Im}(z) = \dfrac{z-\bar{z}}{2i}$.
- Let z, z_1 and z_2 are complex numbers. Then
  - $z = \bar{z} \Leftrightarrow z$ is a real number.
  - $z = -\bar{z} \Leftrightarrow z$ is a purely imaginary number.
  - $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
  - $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
  - $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
  - $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}, z_2 \neq 0$
  - $\overline{(\bar{z})} = z$
  - $\overline{(z^n)} = (\bar{z})^n, n \in I$
  - $\overline{f(z)} = f(\bar{z})$, f being a polynomial with real coefficients.
  - Let $z = x + iy$, then $z\bar{z} = x^2 + y^2$. Thus $z\bar{z}$ is a non-negative real number for any complex number.

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

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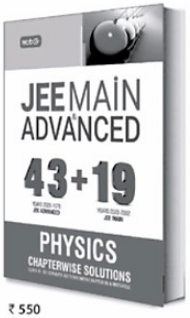

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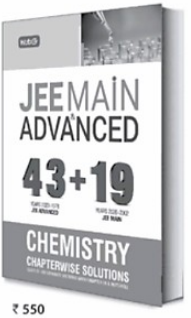

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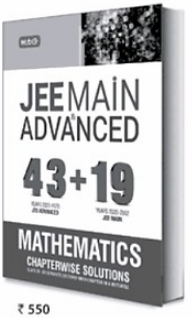

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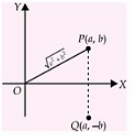
THE MODULUS OF A COMPLEX NUMBER

- Let $z = a + ib$ be a complex number. The modulus of z, denoted by $|z|$, is defined to be a non-negative real number $\sqrt{a^2+b^2}$, *i.e.*, $|z| = \sqrt{a^2+b^2} = (z\bar{z})^{1/2}$.
- Let z, z_1 and z_2 are complex numbers. Then
  - $|z| = |\bar{z}| \geq 0$ and equality holds if and only if $z = 0$.
  - $\text{Re}(z) \leq |z|$ and equality holds if and only if $\text{Im}(z) = 0$ and $\text{Re}(z) \geq 0$.
  - $\text{Im}(z) \leq |z|$ and equality holds if and only if $\text{Re}(z) = 0$ and $\text{Im}(z) \geq 0$.
  - $z\bar{z} = |z|^2$
  - $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$
  - $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1\bar{z}_2)$
  - $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
  - $|z_1 z_2| = |z_1| \cdot |z_2|$
  - $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0$

GEOMETRICAL REPRESENTATION OF COMPLEX NUMBERS

- A complex number $a + ib$ corresponds to the ordered pair (a, b), which can be represented geometrically as the unique point (a, b) in the xy-plane and vice-versa.



- The xy-plane having a complex number assigned to each of its points is called the complex plane or the Argand plane.

  Let $z = a + ib$. Then z is represented as the point $P(a, b)$ on the Argand plane. Obviously $OP = \sqrt{a^2+b^2} = |z|$. Hence $|z|$ is the distance of z from origin. Further if $Q(a, -b)$ is the mirror image of $P(a, b)$ in the x-axis (also called as real axis), then Q is the geometrical representation of $\bar{z}$. Also note that a complex number is real if and only if it lies on the real axis and a complex number is purely imaginary if and only if it lies on the imaginary axis. For any complex numbers z_1 and z_2, it is easy to see that $|z_1 - z_2|$ is the distance of z_1 from z_2, when z_1 and z_2 are represented as points in the Argand plane.

ARGUMENT OF A COMPLEX NUMBER

- Let z be a non-zero complex number, which corresponds to the point P in the Argand plane. If $|z| = r$, then θ satisfies $x = r\cos\theta$, $y = r\sin\theta$, where $z = x + iy$ and θ is called the argument of the given complex number.
- The angle of the line segment OP with the positive direction of x-axis is θ. The angle can also be taken as $2n\pi + \theta, n \in I$. This angle is called argument or amplitude of the complex number z denoted by $\arg(z)$ or $\text{amp}(z)$. Value of θ having minimum modulus value is called the principal argument of z. Thus for any non-zero $z \in C$, principal $\arg(z) \in (-\pi, \pi]$. Obviously a non-zero complex number z is real if and only if principal argument of z is 0 or π and it is purely imaginary if and only if principal argument of z is $\pm\frac{\pi}{2}$.
- **Properties of Argument of complex numbers**

  Let z be a non-zero complex number and $n \in I$. Then
  - $\arg(-z) = \arg(z) + (2n + 1)\pi$
  - $\arg(z + \bar{z}) = n\pi$, z not being purely imaginary.
  - $\arg(z - \bar{z}) = (4n \pm 1)\frac{\pi}{2}$, z not being purely real.
  - $\arg(\bar{z}) = \arg\left(\frac{z\bar{z}}{z}\right) = \arg\left(\frac{1}{z}\right) + 2n\pi$
  - $\arg(z) + \arg(\bar{z}) = 2n\pi$

POLAR FORM OF A COMPLEX NUMBER

- Let $z = x + iy$ be a given non-zero complex number, which corresponds to the complex point P in the Argand plane. Then $OM = x$, $PM = y$. Let $r = |z|$, then $OP = r$. Let $\theta = \arg(z)$, then $\angle POM = \theta$. Now $x = r\cos\theta$ and $y = r\sin\theta$. Hence $z = x + iy = r(\cos\theta + i\sin\theta)$. This form is called polar form of z, sometimes written in short as $r\text{cis}\theta$. Note that (r, θ) are polar coordinates of z.

DE-MOIVRE'S THEOREM

- We define $e^{i\theta} = \cos\theta + i\sin\theta$.

  Let n be any integer, then $(\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos n\theta + i\sin n\theta$

  This result is called De-Moivre's Theorem for an integer. However if $n = \frac{p}{q}, p, q \in I, p \neq 0$ and $q > 1$, then $(\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{p/q} = \cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$

which is one of the q values of $(\cos\theta + i\sin\theta)^{p/q}$.
To make this point more clear consider $(-1)^{1/2}$ *i.e.*, all the complex numbers whose square is -1, which are i and $-i$, for $-1 = \cos\pi + i\sin\pi$.
Hence

$$(-1)^{1/2} = (\cos\pi + i\sin\pi)^{1/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i,$$

which is one of the value of $(-1)^{1/2}$.
For the other value we write $-1 = \cos 3\pi + i\sin 3\pi$

$$\Rightarrow (-1)^{1/2} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i.$$

- **Applications of De-Moivre's Theorem to Trigonometry**
  - Let n be a positive integer. Then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. Also expanding $(\cos\theta + i\sin\theta)^n$ using Binomial theorem and comparing the real and imaginary parts we get

$$\cos n\theta = \cos^n\theta - {}^nC_2\cos^{n-2}\theta\sin^2\theta + {}^nC_4\cos^{n-4}\theta\sin^4\theta - .....$$

$$\sin n\theta = {}^nC_1\cos^{n-1}\theta\sin\theta - {}^nC_3\cos^{n-3}\theta\sin^3\theta + ...$$

Hence, $\tan n\theta = \dfrac{{}^nC_1\tan\theta - {}^nC_3\tan^3\theta + ...}{1 - {}^nC_2\tan^2\theta + {}^nC_4\tan^4\theta - .....}$

- Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$$z_1z_2 = r_1e^{i\theta_1}\cdot r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1+\theta_2)}$$
$$= r_1r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Hence, $|z_1z_2| = r_1r_2 = |z_1|\cdot|z_2|$ and

$$\arg(z_1z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$$

Note:

(i) $\arg(z_1z_2) = \arg(z_1) + \arg(z_2)$

(ii) $\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

(iii) $\arg(z^n) = n\cdot\arg(z) + 2k\pi,\ k \in I$

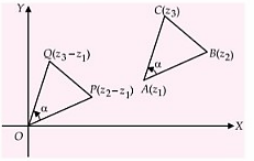
CONCEPT OF ROTATION

- Let z_1, z_2 and z_3 be the vertices of a triangle ABC described in the anticlockwise sense, as shown in the figure.
  Let P and Q be two points in the Argand diagram representing $z_2 - z_1$ and $z_3 - z_1$ respectively. Then ΔOPQ and ΔABC are congruent as

$AB = |z_2 - z_1| = OP$, $AC = |z_3 - z_1| = OQ$ and $BC = |z_3 - z_2| = |(z_3 - z_1) - (z_2 - z_1)| = PQ$.

Hence, $\angle POQ = \alpha$

Now, $\arg\left(\dfrac{z_3 - z_1}{z_2 - z_1}\right) = \arg(z_3 - z_1) - \arg(z_2 - z_1) = \alpha$



Also, $\left|\dfrac{z_3 - z_1}{z_2 - z_1}\right| = \dfrac{AC}{AB}$. Therefore the polar form of $\dfrac{z_3 - z_1}{z_2 - z_1}$ is $\dfrac{AC}{AB}(\cos\alpha + i\sin\alpha)$.

Condition for the four points to be Concyclic

- Let z_1, z_2, z_3 and z_4 be four concyclic points, represented by A, B, C and D in the Argand plane. As the points A, B, C and D are concyclic, so $\angle ADB = \angle ACB$

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$$

(Using the concept of rotation)

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\times\frac{z_1 - z_3}{z_2 - z_3}\right) = 0$$

$\Rightarrow \dfrac{(z_2 - z_4)(z_1 - z_3)}{(z_1 - z_4)(z_2 - z_3)}$ is a positive real number.

Note : The converse of the above result is also true, *i.e.*, if a complex number of the form $\dfrac{(z_2 - z_4)(z_1 - z_3)}{(z_1 - z_4)(z_2 - z_3)}$ is a positive real number, then the points represented by z_1, z_2, z_3 and z_4 in the Argand plane are concyclic.

CUBE ROOTS OF UNITY

- Let $x = \sqrt[3]{1} \Rightarrow x^3 - 1 = 0 \Rightarrow (x-1)(x^2 + x + 1) = 0$

$$\Rightarrow x = 1, \frac{-1 - i\sqrt{3}}{2}, \frac{-1 + i\sqrt{3}}{2}$$

$\Rightarrow x = 1, \omega, \omega^2$, where $\omega = \dfrac{-1 - i\sqrt{3}}{2}$ then $\omega^2 = \dfrac{-1 + i\sqrt{3}}{2}$

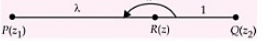
$$\Rightarrow x = 1,\ e^{\frac{2\pi i}{3}},\ e^{\frac{4\pi i}{3}}$$

- **Properties of Cube Roots of Unity**
  - $1+\omega^r+\omega^{2r}=\begin{cases}0, & \text{if } r \text{ is not a multiple of } 3\\ 3, & \text{if } r \text{ is a multiple of } 3\end{cases}$
  - $\omega^{3r+1}=\omega,\ \omega^{3r+2}=\omega^2,\ \omega^{3r}=1$ therefore $\omega^{3r}+\omega^{3r+1}+\omega^{3r+2}=1+\omega+\omega^2=0$
- **Properties of n, n^{th} roots of unity**

  n^{th} roots of unity are $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ where $\alpha=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}, n\in Q.$
  - If n, n^{th} roots of unity are represented in the Argand plane, then they form the vertices of a regular n-gon having centre at origin and circumcircle of this n-gon is $|z|=1$, as modulus value of every root is 1.
  - $1+\alpha+\alpha^2+\ldots+\alpha^{n-1}=0$
  - $1+\alpha^p+(\alpha^p)^2+\ldots+(\alpha^p)^{n-1}=0$ if G.C.D. $(p, n)=1$.
  - $1\cdot\alpha\cdot\alpha^2\ldots\alpha^{n-1}=(-1)^{n-1}$
  - $z^n-1\equiv(z-1)(z-\alpha)(z-\alpha^2)\ldots(z-\alpha^{n-1})$

THE SECTION FORMULA

- Let $P(z_1)$ and $Q(z_2)$ be two given points. Let $R(z)$ be the point which divides the join of P and Q internally in $\lambda:1$ as shown in the figure.



Using the concept of rotation at the point R, we get

$$\frac{z-z_1}{z-z_2}=\lambda e^{i\pi} \Rightarrow z=\frac{z_1+\lambda z_2}{1+\lambda}$$

If R divides PQ externally in $\lambda:1$, then it can be shown that $z=\frac{z_1-\lambda z_2}{1-\lambda}$.

EQUATION OF STRAIGHT LINE IN COMPLEX FORM

- Equation of line through the points z_1 and z_2 in parametric form is given by
$$z=z_2+\lambda(z_1-z_2),\ \lambda\in R$$
- Equation of line through z_1 and z_2 is given by
$$\frac{z-z_1}{z_2-z_1}=\frac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_1} \text{ or } \begin{vmatrix} z & \bar{z} & 1\\ z_1 & \bar{z}_1 & 1\\ z_2 & \bar{z}_2 & 1\end{vmatrix}=0$$
- General equation of a line is given by $\bar{a}z+a\bar{z}+b=0$ where a is a complex number and b is a real number.

Real slope of this line is given by $-\frac{\mathrm{Re}(a)}{\mathrm{Im}(a)}$ and complex slope is given by $-\frac{a}{\bar{a}}=-\frac{\text{coeff. of }\bar{z}}{\text{coeff. of }z}$. Its distance from a point z_1 is given by $\left|\frac{\bar{a}z_1+a\bar{z}_1+b}{2|a|}\right|$.

EQUATION OF A CIRCLE IN COMPLEX FORM

- The equation of the circle having centre at z_0 and radius r is given by $|z-z_0|=r$.
- The general equation of circle is $z\bar{z}+\bar{a}z+a\bar{z}+b=0$, where a is a complex number and b is a real number. The centre of this circle is $-a$ and radius is $\sqrt{|a|^2-b}$.
- The equation of the circle described on the line segment joining z_1 and z_2 as diameter is given by $(z-z_1)(\bar{z}-\bar{z}_2)+(z-z_2)(\bar{z}-\bar{z}_1)=0$.
- $\left|\frac{z-z_1}{z-z_2}\right|=\lambda, \lambda>0, \lambda\neq1$ represents a circle having diameter AB, when A and B divide the join of z_1 and z_2 in $\lambda:1$ internally and externally respectively.
- $\arg\left(\frac{z-z_1}{z-z_2}\right)=\theta$ represents an arc of the circle through z_1 and z_2.
- $|z-z_1|^2+|z-z_2|^2=k$ represents a circle provided $k>\frac{1}{2}|z_1-z_2|^2$.

EQUATION OF CONIC SECTION IN COMPLEX FORM

- Equation of parabola with focus at z_1 and directrix $a\bar{z}+\bar{a}z+b=0$ is $|z-z_1|=\left|\frac{a\bar{z}+\bar{a}z+b}{2|a|}\right|$.
- Equation of ellipse with foci at z_1 and z_2 and length of major axis λ is $|z-z_1|+|z-z_2|=\lambda$.
- Equation of hyperbola with foci at z_1 and z_2 and length of transverse axis λ is $\|z-z_1|-|z-z_2\|=\lambda$.

Monthly Test Drive-8 CLASS XII ANSWER KEY

1. (c)	2. (b)	3. (a)	4. (b)	5. (a)
6. (a)	7. (a,c,d)	8. (a,b)	9. (a,c)	10. (a,d)
11. (b,c)	12. (b,d)	13. (a,b)	14. (a)	15. (b)
16. (b)	17. (2)	18. (8)	19. (6)	20. (32)

PROBLEMS

Single Correct Answer Type

1. Let $a, b \in R, \sqrt{a}\sqrt{b} = \sqrt{ab}$ is not true if
(a) $a \geq 0,\ b \geq 0$ (b) $a \geq 0,\ b < 0$
(c) $a < 0,\ b \geq 0$ (d) $a < 0,\ b < 0$

2. If $z = (\lambda + 3) + i\sqrt{3 - \lambda^2}$, where λ is a real number, then locus of z is
(a) a straight line (b) a circle
(c) a parabola (d) an ellipse

3. Let $ax^2 + bx + c = 0$ be a quadratic equation, whose at least one coefficient is an imaginary number. Then the roots of the equation are
(a) both real
(b) both imaginary and non-conjugate of each other
(c) both imaginary and conjugate of each other
(d) such that at least one of them is imaginary

4. Let z_1 and z_2 be the complex numbers given by $x_1 + iy_1$ and $x_2 + iy_2$, which can also be represented by ordered pairs (x_1, y_1) and (x_2, y_2). If we define $z_1z_2 = (x_1x_2, y_1y_2)$, then which of the following does not become wrong?
(a) $z_1z_2 = z_2z_1$ (b) $1 \cdot z_1 = z_1$
(c) $z_1z_2 = 0 \Rightarrow z_1 = 0$ or $z_2 = 0$
(d) $|z_1z_2| = |z_1| \cdot |z_2|$

5. The trigonometric form of the complex number $z = 1 + i\tan\alpha$, where $\frac{\pi}{2} < \alpha < \pi$, is
(a) $\frac{1}{\cos\alpha}(\cos\alpha + i\sin\alpha)$
(b) $\frac{1}{\cos\alpha}(\cos(-\alpha) + i\sin(-\alpha))$
(c) $\frac{1}{\cos\alpha}(\cos(\pi - \alpha) + i\sin(\pi - \alpha))$
(d) $-\frac{1}{\cos\alpha}[\cos(\pi + \alpha) + i\sin(\pi + \alpha)]$

6. If z be complex number such that equation $|z - a^2| + |z - 2a| = 3$ always represents an ellipse, then range of values of a ($\in R^+$) is
(a) $(1, \sqrt{2})$ (b) $[1, \sqrt{3}]$ (c) $(-1, 3)$ (d) $(0, 3)$

7. If $0 < \alpha < \frac{\pi}{2}$, the modulus and argument form of $(1 + \cos 2\alpha) + i \sin 2\alpha$ is
(a) $-2\cos\alpha[\cos(\pi + \alpha) + i\sin(\pi + \alpha)]$
(b) $2\cos\alpha[\cos\alpha + i\sin\alpha]$
(c) $2\cos\alpha[\cos(-\alpha) + i\sin\alpha(-\alpha)]$
(d) $-2\cos\alpha[\cos(\alpha - \pi) + i\sin(\alpha - \pi)]$

8. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate of each other for ($n \in I$) $x =$
(a) $n\pi$ (b) 0
(c) $(2n+1)\frac{\pi}{2}$ (d) no value of x

9. Find the equation of the circle which touches the line $iz + \bar{z} + 1 + i = 0$ and has the lines $(1 - i)z = (1 + i)\bar{z}$ and $(1 + i)z - (1 - i)\bar{z} = 4i$ as its normals.
(a) $|z - (1 - i)| = \sqrt{2}$ (b) $|z - (1 + i)| = \frac{1}{\sqrt{2}}$
(c) $|z + (1 + i)| = \frac{1}{\sqrt{2}}$ (d) None of these

10. Find all complex numbers z which satisfy $|z|^2 - 2iz + 2c(1 + i) = 0$, where c is real.
(a) $z = c + i\left(-1 \pm \sqrt{1 - 2c - c^2}\right)$
(b) $z = c - i\left(-1 \pm \sqrt{1 + 2c + c^2}\right)$
(c) $z = c + i\left(1 \pm \sqrt{1 + 2c + c^2}\right)$
(d) $z = c - i\left(1 \pm \sqrt{1 - 2c - c^2}\right)$

11. If two consecutive vertices of a regular hexagon be z and $\bar{z}$, find the other vertices if the centre is at $z = 0$.
(a) $ze^{ki\pi/3}$, $k = 1, 2, ..., 5$ (b) $ze^{2ki\pi/3}$, $k = 1, 2, ..., 5$
(c) $ze^{ki\pi/6}$, $k = 1, 2, ..., 5$ (d) $ze^{2ki\pi/5}$, $k = 1, 2, ..., 5$

12. Consider the complex numbers $A(z_1)$, $B(z_2)$ and $C(z_3)$ as the vertices of a triangle ABC. Let P be the foot of altitude from A on BC. Find the complex number associated with the point P.
(a) $\frac{1}{2}\left(\frac{(\bar{z}_1 - \bar{z}_2)(z_2 - z_3)}{(\bar{z}_2 - \bar{z}_3)} + z_1 + z_2\right)$
(b) $\left(\frac{(\bar{z}_1 - \bar{z}_2)(z_2 + z_3)}{(\bar{z}_2 + \bar{z}_3)} + z_1 + z_2\right)$
(c) $\frac{1}{2}\left(\frac{(\bar{z}_1 + \bar{z}_2)(z_2 - z_3)}{\bar{z}_1 + (-\bar{z}_3)} - z_1 - z_2\right)$
(d) $\left(\frac{(\bar{z}_1 + \bar{z}_2)(z_2 - z_3)}{(\bar{z}_2 - \bar{z}_3)} - z_1 - z_2\right)$

13. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$, then the set of possible values of $|z|$ is
(a) {1, 2} (b) {1} (c) {1, 2, 3} (d) {1, 2, 3, 4}

14. Let A, B and C represent the complex numbers z_1, z_2 and z_3 in the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

(a) $\frac{1}{4}(z_1+z_2+z_3)$ (b) $\frac{1}{3}(z_1+z_2+z_3)$

(c) $\frac{1}{2}(z_1+z_2+z_3)$ (d) $z_1+z_2+z_3$

15. $\left(\frac{1+i\tan\theta}{1-i\tan\theta}\right)^n-\frac{1+i\tan n\theta}{1-i\tan n\theta}=$

(a) 0 (b) 1 (c) -1 (d) i

16. Let $\alpha_0, \alpha_1, \alpha_2, \ldots\ldots, \alpha_{n-1}$ be n, n^{th} roots of unity. Value of $(1-\alpha_1)\ldots\ldots(1-\alpha_{n-1})$ is

(a) n (b) $n-1$ (c) $(-n)^n$ (d) 0

17. If $z=x+iy$, then the equation $\left|\frac{2z-i}{z+1}\right|=m$ does not represent a circle, when

(a) $m=1/2$ (b) $m=1$

(c) $m=2$ (d) $m=3$

18. If one root of $z^2+(a+i)z+b+ic=0$ is real, where $a, b, c\in R$, then $c^2+b-ac=$

(a) 0 (b) 1 (c) -1 (d) $a+b+c$

19. If $x=1+i\sqrt{3}$, $y=1-i\sqrt{3}$ and $z=2$, then for any prime $p>3$, $x^p+y^p=$

(a) z^{2p} (b) z^{p-1} (c) z^p (d) z^{p+1}

20. If $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}+\frac{1}{d+\omega}=\frac{2}{\omega}$, where a, b, c, d are real and ω is a non real cube root of unity, then $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}+\frac{1}{d+\omega^2}=$

(a) 2 (b) $2\omega^2$

(c) $\frac{2}{\omega^2}$ (d) None of these

21. Let z and ω be two complex numbers such that $|z|\le 1, |\omega|\le 1$, then $(|z|-|\omega|)^2+(\arg z-\arg\omega)^2\ge$

(a) $|z-\omega|^2$ (b) $|z+\omega+1|^2$

(c) $|z-\omega-1|^2$ (d) None of these

22. If $\left|\frac{z_1-iz_2}{z_1+iz_2}\right|=1$, then $\frac{z_1}{z_2}$ is

(a) purely imaginary (b) real

(c) imaginary (d) of unit modulus

23. The polar form of the complex number $\frac{-16}{1+i\sqrt{3}}$ is

(a) $\frac{8}{\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}}$ (b) $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

(c) $\frac{8}{\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}}$ (d) $8\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$

24. If z, $-z$ and $1-z$ are the vertices of an equilateral triangle, then $\mathrm{Re}(z)=$

(a) 1 (b) 1/2 (c) 1/3 (d) 1/4

25. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c=(1-r)a+rb$ and $w=(1-r)u+rv$, where r is a complex number, then the two triangles

(a) have the same area (b) are similar

(c) are congruent

(d) have the same perimeter

26. The complex number $3+4i$ is rotated about origin by an angle of $\pi/4$ and then stretched 2-times. The complex number corresponding to new position is

(a) $\sqrt{2}(-3+4i)$ (b) $\sqrt{2}(-1+7i)$

(c) $\sqrt{2}(3-4i)$ (d) $\sqrt{2}(-1-7i)$

27. The locus of the point z for which $2\arg\left(\frac{z-i+3}{z+3i-1}\right)=\pi$ is

(a) a straight line passing through the points $3-i$ and $-1+3i$

(b) a straight line passing through the points $-3+i$ and $1-3i$

(c) a circle passing through the points $-3+i$ and $1+3i$

(d) a circle with its centre at the points $-1-i$ and radius $2\sqrt{2}$

28. If $|z-i|\le 2$ and $z_0=5+3i$, then the maximum value of $|iz+z_0|$ is

(a) $2+\sqrt{34}$ (b) $-2+\sqrt{34}$

(c) 5 (d) 7

29. If $|z|=1$, z is non real and $z=\frac{c+i}{c-i}$, where c is real, then the value of c is

(a) $\tan\left[\frac{1}{2}\arg(z)\right]$ (b) $\cot\left(\frac{1}{2}\arg(z)\right)$

(c) $\tan[\arg(z)]$ (d) $\cos[\arg(z)]$

30. If $\frac{a}{|z_1-z_2|}=\frac{b}{|z_2-z_3|}=\frac{c}{|z_3-z_1|}$ $(a, b, c \in R)$, then

$\frac{a^2}{z_1-z_2}+\frac{b^2}{z_2-z_3}+\frac{c^2}{z_3-z_1}$ is

(a) independent of a, b, c only
(b) independent of z_1, z_2, z_3 only
(c) independent of all a, b, c, z_1, z_2, z_3
(d) 1

Multiple Correct Answer Type

31. A non-zero complex number z is uniquely determined if
(a) $\text{Re}(z)$ and $\text{Im}(z)$ are given
(b) $\text{Re}(z)$ and $\arg(z)$ are given
(c) $\text{Im}(z)$ and $|z|$ are given
(d) $|z|$ and $\arg(z)$ are given

32. If z_1, z_2, z_3 and z_4 are the vertices of a square in that order, then
(a) $z_1+z_3=z_2+z_4$
(b) $|z_1-z_2|=|z_2-z_3|=|z_3-z_4|=|z_4-z_1|$
(c) $|z_1-z_3|=|z_2-z_4|$
(d) $\frac{z_1-z_3}{z_2-z_4}$ is purely imaginary

33. If α is a root of $z^5+z^3+z+3=0$, then
(a) $|\alpha| \geq 1$ (b) $|\alpha|<1$
(c) α lies on or outside the unit circle $|z|=1$
(d) α lies inside the unit circle $|z|=1$

34. If $2\cos\theta=x+\frac{1}{x}$ and $2\cos\phi=y+\frac{1}{y}$, then

(a) $x^n+\frac{1}{x^n}=2\cos(n\theta)$ (b) $\frac{x}{y}+\frac{y}{x}=2\cos(\theta-\phi)$

(c) $xy+\frac{1}{xy}=2\cos(\theta+\phi)$

(d) $|x|=|y|=1$

35. If z_1 lies on $|z|=1$ and z_2 lies on $|z|=2$, then
(a) $3 \leq |z_1-2z_2| \leq 5$ (b) $1 \leq |z_1+z_2| \leq 3$
(c) $|z_1-3z_2| \geq 5$ (d) $|z_1-z_2| \geq 1$

36. If complex numbers z_1, z_2, z_3 are the vertices of a triangle ABC then the complex number z which makes the triangle a parallelogram, is given by
(a) $z_1+z_2-z_3$ (b) $z_1+z_3-z_2$
(c) $z_2+z_3-z_1$ (d) $z_1+z_2+z_3$

37. Let $n \in N$ and ω being an imaginary cube root of unity, then $1+\omega^n+\omega^{2n}=$
(a) 0, if n is a multiple of 3
(b) 3, if n is a multiple of 3
(c) 0, if n is not a multiple of 3
(d) 3, if n is not a multiple of 3

38. If $(1+i)z=(1-i)\bar{z}$, then for some $t \in R$, $z=$
(a) $t(1-i)$ (b) $t(1+i)$
(c) $t(-1+i)$ (d) $t(-1-i)$

39. $|z-i\,\text{Re}(z)|=|z-\text{Im}(z)|$, if
(a) $\text{Im}(z)=2, \text{Re}(z)=1$ (b) $\text{Re}(z)=\text{Im}(z)$
(c) $\text{Re}(z)=2, \text{Im}(z)=1$ (d) $\text{Re}(z)+\text{Im}(z)=0$

40. If α is the fifth root of unity, then
(a) $|1+\alpha+\alpha^2+\alpha^3+\alpha^4|=0$
(b) $|1+\alpha+\alpha^2+\alpha^3|=1$
(c) $|1+\alpha+\alpha^2|=2\cos\frac{\pi}{5}$
(d) $|1+\alpha|=2\cos\frac{\pi}{10}$

41. Let z_1 and z_2 be the complex numbers such that $|z_1+iz_2|=|z_1-i\bar{z}_2|$. Then
(a) z_1 lies on real axis (b) z_1 lies on imaginary axis
(c) z_2 lies on real axis (d) z_2 lies on imaginary axis

42. The value of $\sum_{k=1}^{n}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is
(a) -1, for $n=6$ (b) 0, for $n=7$
(c) i, for $n=6$ (d) $-i$, for $n=7$

43. Number of roots of the equation $z^{10}-z^5-992=0$ with negative real part is
(a) 3 (b) 4
(c) 5 (d) an even number

44. Equation of the reflection of the line $\bar{a}z+a\bar{z}=0$ in the real axis is
(a) $\overline{az}+az=0$ (b) $\bar{a}z-a\bar{z}=0$
(c) $(a+\bar{a})(z+\bar{z})=0$ (d) $(a-\bar{a})(z+\bar{z})=0$

45. Equation of line through a and ib, $a, b \in R$, $a, b \neq 0$ in complex form is

(a) $z\left(\frac{1}{2a}-\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}+\frac{i}{2b}\right)=1$

(b) $z\left(\frac{1}{2a}+\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}+\frac{i}{2b}\right)=1$

(c) $z\left(\frac{1}{2a}-\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}-\frac{i}{2b}\right)=1$

(d) $z\left(\frac{1}{2a}+\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}-\frac{i}{2b}\right)=1$

46. Given that the two curves $\arg(z)=\frac{\pi}{6}$ and $|z-2\sqrt{3}i|=r$ intersect in two distinct points, then

(a) $r>3$ (b) $r=6$

(c) $0<r<3$ (d) $[r]\neq 2$

47. If $\arg\left(\frac{z_1+z_2}{z_1-z_2}\right)=\frac{\pi}{2}$ and $|z_1+z_2|\neq|z_1-z_2|$, then the quadrilateral formed by 0, z_1, z_2 and z_1+z_2 is a

(a) rhombus (b) parallelogram

(c) square (d) rectangle

48. If z satisfies $|z-1-2i|\leq 1$, then

(a) $\min(\arg z)=\tan^{-1}\frac{1}{4}$ (b) $\max(\arg z)=\frac{\pi}{2}$

(c) $\min|z|=\sqrt{5}-1$ (d) $\max|z|=\sqrt{5}+1$

Comprehension Type

Paragraph for Question No. 49 to 51

Suppose $z=x+iy$ is identified with the matrix $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$.

49. Which of the following is wrong?

(a) $z=0\Leftrightarrow x=y=0$ (b) $2z=z+z$

(c) $z\bar{z}=|z|^2$ (d) None of these

50. What does the symmetric of this matrix mean in terms of a complex number z?

(a) $z=0$ (b) $\mathrm{Re}(z)=0$

(c) $\mathrm{Im}(z)=0$ (d) $\mathrm{Re}(z)=\mathrm{Im}(z)$

51. What does the skew symmetric of this matrix mean in terms of a complex number z?

(a) $z=0$ (b) $\mathrm{Re}(z)=0$

(c) $\mathrm{Im}(z)=0$ (d) $\mathrm{Re}(z)=-\mathrm{Im}(z)$

Paragraph for Question No. 52 to 54

Suppose z and w be two complex numbers such that $|z|\leq 1$, $|w|\leq 1$ and $|z+iw|=|z-i\bar{w}|=2$. Use the results $|z|^2=z\bar{z}$ and $|z+w|\leq|z|+|w|$, answer the following questions.

52. Which of the following is true about $|z|$ and $|w|$?

(a) $|z|=|w|=\frac{1}{2}$ (b) $|z|=\frac{1}{2}$, $|w|=\frac{3}{4}$

(c) $|z|=|w|=\frac{3}{4}$ (d) $|z|=|w|=1$

53. Which of the following is true for z and w?

(a) $\mathrm{Re}(z)=\mathrm{Re}(w)$ (b) $\mathrm{Im}(z)=\mathrm{Im}(w)$

(c) $\mathrm{Re}(z)=\mathrm{Im}(w)$ (d) $\mathrm{Im}(z)=\mathrm{Re}(w)$

54. Number of complex numbers z satisfying the above conditions is

(a) 1 (b) 2 (c) 4 (d) infinite

Matrix-Match Type

55. Match for centre of circles in Column-I to Column-II.

	Column–I		Column–II
A.	$\lvert z-2\rvert^2+\lvert z-4i\rvert^2=20$	p.	$1-i$
B.	$\left\lvert\frac{z-1}{z+1}\right\rvert=\frac{1}{2}$	q.	$\frac{5}{3}+0i$
C.	$z\bar{z}-(1+i)z-(1-i)\bar{z}+7=0$	r.	$-4-i$
D.	$\arg\left(\frac{z+3+4i}{z+5-2i}\right)=\frac{\pi}{2}$	s.	$1+2i$

Numerical Answer Type

56. If the equation $z^2-(3+i)z+m+2i=0$, $m\in R$ has a real root, then the value of m is _____.

57. If 2 and i are two roots of the equation $2z^4+az^3+bz^2+cz+3=0$, where $a, b, c\in R$, then the value of $-2a/11$ is _____.

58. If the equation $x^4+ax^3+bx^2+cx+d=0$ ($a, b, c, d\in R$) has four imaginary roots, two with sum $3+4i$ and the other two with product $13+i$, then the value of $b/17$ is _____.

59. Let z_1 and z_2 be two complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular. If z_2 is not unimodular then $|z_1|$ is _____.

60. If $1, \alpha, \alpha^2, \alpha^3$ and α^4 are the roots of $z^5=1$, then the value of $1+\alpha+\alpha^2+\frac{1}{\alpha^2}+\frac{1}{\alpha}$ is _____.

61. If $z^2-z+1=0$, then the value of

$$\frac{1}{12}\left[\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^{24}+\frac{1}{z^{24}}\right)^2\right]$$

is equal to _____.

62. If r_1 and r_2 are the maximum and minimum distances of a point on the curve $10(z\bar{z})-3i\{z^2-(\bar{z})^2\}-16=0$ from origin, then value of r_1+r_2 is _____.

63. Let γ be the imaginary part of

$(z-1)(\cos\alpha+i\sin\alpha)^{-1}+(z-1)^{-1}(\cos\alpha+i\sin\alpha)$

where $z(\neq 1)$ is complex and α is real. If $\gamma = 0$ then z lies on a circle whose radius is equal to _____.

64. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ then $|z_1 + z_2 + z_3|$ is equal to _____.

65. The number of complex numbers which are conjugate of their own cube, is _____.

SOLUTIONS

1. (d) : If $a = -4$, $b = -9$, then $\sqrt{a}\sqrt{b} = (2i)(3i) = -6$, while $\sqrt{ab} = \sqrt{36} = 6$.

2. (b) : Let $z = x + iy$, then $x = \lambda + 3$ and $y = \sqrt{3-\lambda^2}$

$\Rightarrow \quad y^2 = 3-(x-3)^2 \quad \Rightarrow \quad (x-3)^2 + y^2 = 3,$

which represents a circle.

3. (d) : Obviously (a) and (c) are not true as in this case all a, b, c are real, which is not the case. However (b) is also not necessarily true, as the roots of the equation $x^2 + ix = 0$ are 0 and $-i$.

4. (a) : According to the new definition

$1 \cdot z_1 = (1, 0) \cdot (x, y) = (x, 0)$

$(1, 0) \cdot (0, 1) = (0, 0)$ without being $(1, 0) = (0, 0)$ or $(0, 1) = (0, 0)$

$|z_1z_2| = \sqrt{(x_1x_2)^2 + (y_1y_2)^2} \neq \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}$

Thus, (b), (c) and (d) are wrong.

5. (d) : $|z| = |\sec\alpha| = -\sec\alpha$, as $\frac{\pi}{2} < \alpha < \pi$, and $\arg(z) = \alpha - \pi$.

Also, $\cos(\alpha-\pi) = \cos(\alpha+\pi)$

and $\sin(\alpha-\pi) = \sin(\alpha+\pi)$

6. (d) : $|a^2 - 2a| < 3 \Rightarrow a \in (0, 3)$

7. (b) : $1 + \cos 2\alpha + i\sin 2\alpha = 2\cos\alpha(\cos\alpha + i\sin\alpha)$

8. (d) : $\sin x - i\cos 2x = \cos x - i\sin 2x$

$\Rightarrow \sin x = \cos x$ and $\cos 2x = \sin 2x$

which is not true for any value of x.

9. (b) : Clearly, point of intersection of normals would be the centre of the required circle.

$\because \quad (1-i)z = (1+i)\bar{z} \Rightarrow \bar{z} = \dfrac{1-i}{1+i}z \qquad ...(i)$

and $(1+i)z - (1-i)\bar{z} = 4i \Rightarrow \bar{z} = \dfrac{4i-(1+i)z}{i-1} \qquad ...(ii)$

From (i) and (ii), $(1-i)(i-1)z = 4i(1+i) - (1+i)^2 z$

$\Rightarrow z(4i) = 4i(1+i) \Rightarrow z = (1+i)$

Now, equation of tangent can be rewritten as,

$\dfrac{1}{1+i}(iz+\bar{z}) + 1 = 0 \Rightarrow i(1-i)z + (1-i)\bar{z} + 2 = 0$

$\Rightarrow (i+1)z + (1-i)\bar{z} + 2 = 0$

Now, distance of $z = (1 + i)$ from this line

$= \dfrac{|(1+i)(1+i) + (1-i)(1-i) + 2|}{2|1-i|} = \dfrac{1}{\sqrt{2}}$

Thus, the equation of required circle is

$|z-(1+i)| = \dfrac{1}{\sqrt{2}}$

10. (a) : Put $z = a + ib$ then $a^2 + b^2 - 2ai + 2b + 2c + 2ci = 0$

$\Rightarrow (a^2 + b^2 + 2b + 2c) + (2c - 2a)i = 0$

$\Rightarrow a^2 + b^2 + 2b + 2c = 0$ and $2c - 2a = 0 \Rightarrow a = c$

Now, $b^2 + 2b + (c^2 + 2c) = 0 \Rightarrow b = -1 \pm \sqrt{1-2c-c^2}$

Since b is real, so, $1 - 2c - c^2 \geq 0$

$\Rightarrow c \in [-1-\sqrt{2},\ -1+\sqrt{2}]$

$\Rightarrow z = c + i(-1 \pm \sqrt{1-2c-c^2})$

11. (b) : $\bar{z} = ze^{i\pi/3}$. Let $z = e^{i\theta}$ then $\theta = -\dfrac{\pi}{6}$.

So, $z = r\left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\right)$

The remaining vertices are $ze^{2ki\pi/3}, k = 1, 2, ..., 5$.

12. (a) : $\angle APC = \dfrac{\pi}{2} \Rightarrow \arg\left(\dfrac{z-z_1}{z_2-z_3}\right) = \dfrac{\pi}{2}$ or $-\dfrac{\pi}{2}$

$\Rightarrow \dfrac{z-z_1}{z_2-z_3}$ is purely imaginary

$\Rightarrow \dfrac{z-z_1}{z_2-z_3} + \dfrac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_3} = 0 \Rightarrow \dfrac{\bar{z}}{\bar{z}_2-\bar{z}_3} = \dfrac{\bar{z}_1}{\bar{z}_2-\bar{z}_3} - \dfrac{z-z_1}{z_2-z_3}$

Also $P(z)$ is lying on the line through B and C.

$\therefore \dfrac{z-z_2}{z_2-z_3} = \dfrac{\bar{z}-\bar{z}_2}{\bar{z}_2-\bar{z}_3} \Rightarrow \dfrac{\bar{z}}{\bar{z}_2-\bar{z}_3} = \dfrac{\bar{z}_2}{\bar{z}_2-\bar{z}_3} + \dfrac{z-z_2}{z_2-z_3}$

$\Rightarrow \dfrac{z-z_2}{z_2-z_3} + \dfrac{\bar{z}_2}{\bar{z}_2-\bar{z}_3} = \dfrac{\bar{z}_1}{\bar{z}_2-\bar{z}_3} - \dfrac{z-z_1}{z_2-z_3}$

$\Rightarrow z = \dfrac{1}{2}\left(\dfrac{(\bar{z}_1-\bar{z}_2)(z_2-z_3)}{\bar{z}_2-\bar{z}_3} + z_1 + z_2\right)$

13. (b) : The given equation is $(z^2 + z + 1)(z^2 + 1) = 0$

$z = \pm i, \omega, \omega^2$, ω being an imaginary cube root of unity.

Thus, $|z| = 1$.

14. (d) : Centroid of ΔABC is at $\frac{z_1+z_2+z_3}{3}$.

It is a known fact that orthocentre divides the join of centroid and circumcentre in 2 : 3 externally. So, orthocentre is given by $z_1 + z_2 + z_3$.

15. (a) : The given expression is

$$\left(\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta}\right)^n-\frac{\cos n\theta+i\sin n\theta}{\cos n\theta-i\sin n\theta}=0,$$

as $(\cos\theta-i\sin\theta)^n=(\cos(-\theta)+i\sin(-\theta))^n$

$= \cos n\theta - i \sin n\theta$

16. (a) : $x^n-1\equiv(x-1)(x-\alpha_1)(x-\alpha_2)....(x-\alpha_{n-1})$

$\Rightarrow (x-\alpha_1)(x-\alpha_2)....(x-\alpha_{n-1})=1+x+x^2+....+x^{n-1}$.

Put $x = 1$.

$\therefore\ (1 - \alpha_1)(1 - \alpha_2) \ldots (1 - \alpha_{n-1}) = 1 + 1 + \ldots n$ terms

$\Rightarrow (1 - \alpha_1)(1 - \alpha_2) \ldots (1 - \alpha_{n-1}) = n$

17. (c) : The given equation is $\left|\frac{z-\frac{i}{2}}{z+1}\right|=\frac{m}{2}$, which does not represent a circle when $\frac{m}{2}=1$.

18. (a) : Let $\alpha \in R$ be a root.

Then, $\alpha^2+(a+i)\alpha+b+ic=0$

$\Rightarrow\ \alpha^2+a\alpha+b=0$ and $\alpha + c = 0 \Rightarrow c^2 - ac + b = 0$

19. (c) : Here, $x=-2\omega,\ y=-2\omega^2$; where ω is an imaginary cube root of unity.

Now, $x^p+y^p=-2^p\omega^p[1+\omega^p]$. Now use the fact that p is a prime greater than 3, so p is of the form $6k + 1$ or $6k - 1$, $k \in N$.

20. (c) : If ω is a root of the equation

$\frac{1}{a+x}+\frac{1}{b+x}+\frac{1}{c+x}+\frac{1}{d+x}=\frac{2}{x}$, then ω^2 is also a root.

21. (a) : Let $z=r_1(\cos\theta+i\sin\theta),\ \omega=r_2(\cos\alpha+i\sin\alpha)$

$$|z-\omega|^2=(r_1-r_2)^2-2r_1r_2\cos(\theta-\alpha)+2r_1r_2$$

$$=(r_1-r_2)^2+2r_1r_2(1-\cos(\theta-\alpha))$$

$$=(r_1-r_2)^2+2r_1r_2\,2\sin^2\left(\frac{\theta-\alpha}{2}\right)$$

$$\le (r_1-r_2)^2+4(1)(1)\left(\frac{\theta-\alpha}{2}\right)^2$$

$$\left[\because\ \sin^2\left(\frac{\theta-\alpha}{2}\right)\le\left(\frac{\theta-\alpha}{2}\right)^2\right]$$

$\therefore\ |z-\omega|^2 \le (|z|-|\omega|)^2+(\text{Arg}z-\text{Arg}\omega)^2$

22. (b) : $\left|\frac{\frac{z_1}{z_2}-i}{\frac{z_1}{z_2}+i}\right|=1 \Rightarrow \frac{z_1}{z_2}$ is equidistant from $-i$ and i

$\Rightarrow \frac{z_1}{z_2}$ is a real number.

23. (b) : $\frac{-16}{1+i\sqrt{3}}=-4+i(4\sqrt{3})$, whose modulus value is 8 and argument is $\frac{2\pi}{3}$.

24. (d) : $z^2 + z^2 + (1 - z)^2 = -z^2 - z + z^2 + z - z^2$

$\Rightarrow\ 4z^2 - 2z + 1 = 0 \ \Rightarrow\ z=\frac{2\pm\sqrt{4-16}}{8}$.

Thus, $\text{Re}(z) = \frac{1}{4}$.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

					6	1									2	4				
					9	5									9	7				
7	1									1	9									
4	3			1						2	5			1						
			9		5								3		7					
				3										5				9	6	
						6	5		7										1	7
		2	5						2				4	2						
		1	4					5		6			1	5						
						8			2											
					9	5					1	3								
						1			9											
				2	5			4		3					4	5				
				7	2			1							9	3				
2	9							9		3	7									
1	3		2							2	1		4							
		9		5									1		5					
			3			5	4							3				7	9	
						9	1											6	2	
		9	1										5	2						
		8	2										9	8						

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

25. (b) : $r=\dfrac{c-a}{b-a}=\dfrac{w-u}{v-u}$

$\Rightarrow \arg\left(\dfrac{c-a}{b-a}\right)=\arg\left(\dfrac{w-u}{v-u}\right)$ and $\left|\dfrac{c-a}{b-a}\right|=\left|\dfrac{w-u}{v-u}\right|$.

26. (b) : The new complex number is

$2(3+4i)e^{i\pi/4}=\sqrt{2}(-1+7i)$

27. (d) : $\arg\left(\dfrac{z-(-3+i)}{z-(1-3i)}\right)=\dfrac{\pi}{2}$

$\Rightarrow$ z lies on a circle having diametrically opposite points $-3+i$ and $1-3i$.

28. (d) : $|iz+z_0| = |i(z-i)+4+3i|$

$\le |z-i|+|4+3i| \le 7$.

29. (b) : Let $z=e^{i\theta}$, then

$$\frac{c}{i}=\frac{\cos\theta+1+i\sin\theta}{\cos\theta-1+i\sin\theta}=\frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}{2i\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}=\frac{1}{i}\cot\frac{\theta}{2}$$

$\Rightarrow c=\cot\dfrac{\theta}{2}$

30. (c) : $a^2=\lambda(z_1-z_2)(\bar{z}_1-\bar{z}_2)$ etc. for some $\lambda \in R$.

$\Rightarrow \sum\dfrac{a^2}{z_1-z_2}=0$

31. (a, d) : (a) $z = \mathrm{Re}(z) + i\,\mathrm{Im}(z)$

(b) If $\mathrm{Re}(z) = 0$ and $\arg z = \pi/2$, z cannot be fixed.

(c) $\mathrm{Re}(z)$ has two possible values.

(d) $z = |z|\,(\cos(\arg z) + i\sin(\arg z))$

32. (a, b, c, d) : Let the four points represented by z_1, z_2, z_3 and z_4 be A, B, C and D respectively. Since $ABCD$ is a square, the mid point of AC = the mid point of BD.

$\Rightarrow \dfrac{1}{2}(z_1+z_3)=\dfrac{1}{2}(z_2+z_4) \quad \Rightarrow \quad z_1+z_3=z_2+z_4$

Also, $AB = BC = CD = DA$

$\Rightarrow |z_1-z_2| = |z_2-z_3| = |z_3-z_4| = |z_4-z_1|$

Since diagonals of the square $ABCD$ are equal.

$\therefore \quad AC = BD \quad \Rightarrow \quad |z_1-z_3| = |z_2-z_4|$

Also, since $AC \perp BD$, or $\dfrac{z_1-z_3}{z_2-z_4}$ is purely imaginary.

33. (a, c) : Suppose $|\alpha| < 1$, we have

$|-3| = |\alpha^5+\alpha^3+\alpha| \le |\alpha|^5+|\alpha|^3+|\alpha| < 1+1+1$

$\Rightarrow 3 < 3$, A contradiction

Thus $|\alpha| \ge 1$

$\Rightarrow$ α lies either on or outside the unit circle $|z| = 1$.

34. (a, b, c, d) : $\because 2\cos\theta = x+\dfrac{1}{x} \Rightarrow x=\cos\theta\pm i\sin\theta$

Similarly, $y=\cos\phi\pm i\sin\phi$

$\therefore \quad x^n+\dfrac{1}{x^n}=\cos(n\theta)+i\sin(n\theta)+\cos(n\theta)-i\sin(n\theta)$

$= 2\cos(n\theta)$

Now, $\dfrac{x}{y}=\cos(\theta-\phi)+i\sin(\theta-\phi)$

$\therefore \quad \dfrac{x}{y}+\dfrac{y}{x}=2\cos(\theta-\phi)$

And, $xy=\cos(\theta+\phi)+i\sin(\theta+\phi)$

$\Rightarrow \quad xy+\dfrac{1}{xy}=2\cos(\theta+\phi)$

35. (a, b, c, d) : Let $z_1 = \cos\theta + i\sin\theta$ and $z_2 = 2(\cos\phi + i\sin\phi)$

$\therefore \quad |z_1-2z_2|^2=(\cos\theta-4\cos\phi)^2+(\sin\theta-4\sin\phi)^2$

$= 1 + 16 - 8\cos(\theta-\phi)$

$\Rightarrow 9 \le |z_1-2z_2|^2 \le 25$

Similarly,

$1 \le |z_1+z_2| \le 3, |z_1-z_2| \ge 1$ and $|z_1-3z_2| \ge 5$.

36. (a, b, c) : Complete the parallelogram, taking BC, CA and AB as diagonals.

37. (b, c) : If n is a multiple of 3, then let $n = 3k$.

Thus $1+\omega^n+\omega^{2n}=1+(\omega^3)^k+(\omega^6)^k=3$

If n is not a multiple of 3, then either $n = 3k + 1$ or $n = 3k - 1$, in each case $1+\omega^n+\omega^{2n}=0$

38. (a, c) : Put $z = x + iy$, in the given relation to get $x + y = 0$. Thus x and y are of opposite sign.

39. (b, d) : Let $z = x + iy$,

then $|x+i(y-x)| = |x-y+iy| \Rightarrow x^2 = y^2$

40. (a, b, c) : We have $\alpha=\cos\dfrac{2\pi}{5}+i\sin\dfrac{2\pi}{5}$

and $1+\alpha+\alpha^2+\alpha^3+\alpha^4=0$

$\therefore \quad |1+\alpha+\alpha^2+\alpha^3| = |-\alpha^4| = |\alpha|^4 = 1$

Also $|1+\alpha+\alpha^2| = |-\alpha^3(1+\alpha)| = |1+\alpha|$ ... (i)

$=\left|1+\cos\dfrac{2\pi}{5}+i\sin\dfrac{2\pi}{5}\right|=\left|2\cos\dfrac{\pi}{5}\left(\cos\dfrac{\pi}{5}+i\sin\dfrac{\pi}{5}\right)\right|=2\cos\dfrac{\pi}{5}$

Again from (i), $|1+\alpha| = |1+\alpha+\alpha^2| = 2\cos\dfrac{\pi}{5}$.

41. (a, d) : Given, $|z_1-(-iz_2)| = |z_1-(i\bar{z}_2)|$

$\Rightarrow$ z_1 is equidistant from $-iz_2$ and $i\bar{z}_2$ (conjugate of $-iz_2$). Thus z_1 lies on real axis.

Further given condition implies $|z_2-iz_1| = |z_2-\bar{z}_1 i|$

$\Rightarrow$ z_2 is equidistant from iz_1 and $i\bar{z}_1$

$\Rightarrow$ z_2 lies on imaginary axis.

42. (b, c) : $\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7} = -i\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$

Thus, for $n = 7$, the given sum is zero and for $n = 6$, the given sum is i.

43. (c) : The given equation is $(z^5 - 32)(z^5 + 31) = 0$

$\Rightarrow z = 2(1)^{1/5}$ or $z = \sqrt[5]{31}(-1)^{1/5}$

44. (a) : If α lies on $\bar{a}z + a\bar{z} = 0$, then $\bar{a}\alpha + a\bar{\alpha} = 0$. We would like to see an equation which is satisfied by $\bar{\alpha}$, which is $\bar{a}\bar{z} + az = 0$.

45. (a) : If z lies on the line, then

$$\arg\left(\frac{z-ib}{z-a}\right) = 0 \text{ or } \pi \Rightarrow \frac{z-ib}{z-a} = \frac{\bar{z}+ib}{\bar{z}-a}$$

$\Rightarrow z(-a-ib) + \bar{z}(-ib+a) + 2iab = 0$

$\Rightarrow z\left(-\frac{1}{2ib} - \frac{1}{2a}\right) + \bar{z}\left(-\frac{1}{2a} + \frac{1}{2ib}\right) + 1 = 0$

$\Rightarrow z\left(\frac{1}{2a} + \frac{1}{2ib}\right) + \bar{z}\left(\frac{1}{2a} - \frac{1}{2ib}\right) = 1$

$\Rightarrow z\left(\frac{1}{2a} - \frac{i}{2b}\right) + \bar{z}\left(\frac{1}{2a} + \frac{i}{2b}\right) = 1$

46. (a, b, d) : Let $z = x + iy$;

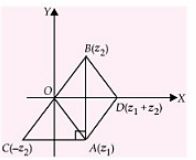
$\arg(z) = \frac{\pi}{6} \Rightarrow \frac{y}{x} = \tan\frac{\pi}{6} \Rightarrow y = \frac{1}{\sqrt{3}}x$, which is a straight line. Also, $|z - 2\sqrt{3}i| = r$, represents a circle with centre at $(0, 2\sqrt{3})$ and radius r. The straight line will intersect the circle if the perpendicular distance from the centre on the line $< r \Rightarrow \left|\frac{0 - 2\sqrt{3}\cdot\sqrt{3}}{2}\right| < r \Rightarrow r > 3$.

Therefore $[r] \geq 3$.

47. (a, b) : According to the given condition ABC is a right angled triangle, right angled at A.

Thus $OA = OB$.

As there are infinitely many positions of A, except at one position $\angle BOA$ is never $\pi/2$. Hence quadrilateral $OADB$ is a rhombus and hence a parallelogram.

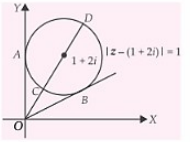


48. (b, c, d) : A is the point for max ($\arg(z)$).

B is the point for min ($\arg(z)$).

C is the point for min ($|z|$).

D is the point for max ($|z|$).



49. (d) : $z = 0 \Rightarrow \begin{vmatrix} x & y \\ -y & x \end{vmatrix} = 0 \Rightarrow x = y = 0$

Converse is also true. It can be seen that (b) and (c) are also true.

50. (c) : If matrix is symmetric, then

$$\begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} x & -y \\ y & x \end{bmatrix} \Rightarrow y = 0 \Rightarrow \text{Im}(z) = 0.$$

51. (b) : If matrix is skew symmetric, then

$$\begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} -x & y \\ -y & -x \end{bmatrix} \Rightarrow x = 0 \Rightarrow \text{Re}(z) = 0.$$

52. (d) : $\because |z + iw| \leq |z| + |iw| = |z| + |i||w| \leq 2$

$\therefore |z + iw| = 2 \Leftrightarrow |z| = |w| = 1$

53. (d) : Let $z = x + iy$ and $w = \alpha + i\beta$

Now, $|z + iw| = 2 \Rightarrow (z + iw)(\bar{z} - i\bar{w}) = 4$

$\Rightarrow |z|^2 + |w|^2 + iw\bar{z} - i\bar{w}z = 4 \Rightarrow iw\bar{z} - i\bar{w}z = 2$ ... (1)

and $|z - i\bar{w}| = 2 \Rightarrow (z - i\bar{w})(\bar{z} + iw) = 4$

$\Rightarrow |z|^2 + |w|^2 + iwz - i\bar{w}\bar{z} = 4 \Rightarrow iwz - i\bar{w}\bar{z} = 2$ ... (2)

Adding (1) and (2), we get

$i(w - \bar{w})(z + \bar{z}) = 4 \Rightarrow i(2i\beta)(2x) = 4 \Rightarrow \beta x = -1$ ... (3)

Subtracting (1) from (2), we get

$i(w + \bar{w})(z - \bar{z}) = 0 \Rightarrow \alpha y = 0$ ... (4)

From (4), either $\alpha = 0$ or $y = 0$

If $y = 0$, then $x^2 + y^2 = 1 \Rightarrow x = \pm 1 \Rightarrow z = 1$ or -1.

If $\alpha = 0$, then $\alpha^2 + \beta^2 = 1 \Rightarrow \beta = \pm 1 \Rightarrow w = \pm i$.

So, $\text{Im}(z) = \text{Re}(w) = 0$.

54. (b) : As $z = \pm 1$, so two values of z can be obtained.

55. A → s; B → q; C → p; D → r

(A) The given equation represents a circle with the segment joining 2 and $4i$ as diameter. Therefore, its centre is $(1 + 2i)$.

(B) $4|z - 1|^2 = |z + 1|^2$

$\Rightarrow 4(z\bar{z} - z - \bar{z} + 1) = z\bar{z} + z + \bar{z} + 1$

$\Rightarrow 3z\bar{z} - 5z - 5\bar{z} + 3 = 0$

It is a circle with centre at $(5/3 + 0i)$.

(C) It represents a circle with centre at $(1 - i)$.

(D) Diameter of the circle is the segment joining $(-3 - 4i)$ and $(-5 + 2i)$. Thus, its centre is $(-4 - i)$.

56. (2) : Let r be the real root.

Then, $r^2-(3+i)r+m+2i=0$.

Now, equating the real and imaginary parts, we get

$r^2-3r+m=0$ and $2-r=0 \Rightarrow r=2$

$\therefore \quad m=6-4=2$

57. (1) : Let the roots be α, 2, i and $-i$.

Then, product of roots $=\frac{3}{2} \Rightarrow \alpha=\frac{3}{4}$

Now, $-\frac{a}{2}=\alpha+2+i-i=\frac{11}{4} \Rightarrow a=-\frac{11}{2}$

58. (3) : Let the roots be α, β, $\bar{\alpha}$ and $\bar{\beta}$.

Given $\alpha+\beta=3+4i$ and $\bar{\alpha}\bar{\beta}=13+i$.

Now, $b=\alpha\beta+\alpha\bar{\alpha}+\alpha\bar{\beta}+\bar{\alpha}\beta+\beta\bar{\beta}+\bar{\alpha}\bar{\beta}$

$=13-i+\alpha(\bar{\alpha}+\bar{\beta})+\beta(\bar{\alpha}+\bar{\beta})+13+i$

$=13-i+(\alpha+\beta)(\bar{\alpha}+\bar{\beta})+13+i = 26+25=51$

59. (2) : Given that $|z_1-2z_2|=|2-z_1\bar{z}_2|$

$\Rightarrow (|z_1|^2-4)(|z_2|^2-1)=0 \Rightarrow |z_1|=2$, as $|z_2|\neq 1$

60. (0) : $1+\alpha+\alpha^2+\frac{1}{\alpha^2}+\frac{1}{\alpha}=1+\alpha+\alpha^2+\alpha^3+\alpha^4=0$

61. (4) : Solving $z^2-z+1=0$, we get

$z=\frac{1\pm i\sqrt{3}}{2}=\cos\frac{\pi}{3}\pm i\sin\frac{\pi}{3}$

$\therefore \left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots$

$\ldots\ldots+\left(z^{24}+\frac{1}{z^{24}}\right)^2=48$

62. (3) : $10(z\bar{z})-3i\{z^2-(\bar{z})^2\}-16=0$

$\Rightarrow \quad 5(x^2+y^2)+6xy-8=0 \qquad \ldots (1)$

Let $(r\cos\theta, r\sin\theta)$ be a point on (1), then

$5r^2+6r^2\sin\theta\cos\theta-8=0 \Rightarrow r^2=\frac{8}{5+3\sin 2\theta}$

Clearly, $1\le r^2\le 4 \Rightarrow 1\le |r|\le 2$.

$\therefore r_1=|r|_{\max}=2$ and $r_2=|r|_{\min}=1 \Rightarrow r_1+r_2=3$.

63. (1) : Let $\beta=\frac{z-1}{\cos\alpha+i\sin\alpha}+\frac{\cos\alpha+i\sin\alpha}{z-1}$

Then, $\beta-\bar{\beta}=0$

$\Rightarrow \frac{\{(z-1)(\bar{z}-1)-1\}(\cos\alpha-i\sin\alpha)}{(\bar{z}-1)}$

$+\frac{\{1-(z-1)(\bar{z}-1)\}(\cos\alpha+i\sin\alpha)}{z-1}=0$

$\Rightarrow \frac{(z-1)(z-1)(\bar{z}-1)(z-1)}{(z-1)(\bar{z}-1)(\bar{z}-1)(\bar{z}-1)}=\frac{\cos\alpha+i\sin\alpha}{\cos\alpha-i\sin\alpha}$

$\Rightarrow |z-1||(z-1)(\bar{z}-1)-1|=|\bar{z}-1||(z-1)(\bar{z}-1)-1|$

$\Rightarrow (z-1)(\bar{z}-1)-1=0$ or $x^2+y^2-2x=0$

which represents equation of a circle whose radius is 1.

64. (2) : $z_1\bar{z}_1=1, z_2\bar{z}_2=4, z_3\bar{z}_3=9$

$\therefore \quad |9z_1z_2+4z_1z_3+z_2z_3|$

$=|z_1z_2z_3\bar{z}_3+z_1z_3z_2\bar{z}_2+z_2z_3z_1\bar{z}_1|=12$

$\Rightarrow |z_1z_2z_3||\bar{z}_1+\bar{z}_2+\bar{z}_3|=12 \Rightarrow 6|z_1+z_2+z_3|=12$

$\Rightarrow \quad |z_1+z_2+z_3|=2$

65. (5) : Let z be such a complex number, then

$z^3=\bar{z} \Rightarrow |z|=0$ or $|z|=1$

$\Rightarrow z=0$ or $z=\cos\theta+i\sin\theta$

If $z=\cos\theta+i\sin\theta$, then $\cos3\theta=\cos\theta$ and $\sin3\theta=-\sin\theta$

$\Rightarrow \quad \sin\theta=0 \quad$ or $\quad \sin\theta=1 \quad$ or $\quad \sin\theta=-1$

$\Rightarrow \quad \theta=0$ or π, $\frac{\pi}{2}$ or $-\frac{\pi}{2}$

or $z=i, 1, -i, -1$ or $z=0$.

SAMURAI SUDOKU

ANSWER · DECEMBER 2020

1	2	5	6	7	8	4	9	3
7	3	8	1	9	4	6	5	2
4	9	6	5	2	3	8	1	7
5	7	4	3	6	1	9	2	8
9	6	3	8	4	2	5	7	1
2	8	1	9	5	7	3	6	4
8	5	7	4	1	9	2	3	6
6	4	2	7	3	5	1	8	9
3	1	9	2	8	6	7	4	5

5	4	9	8	1	7	9	4	5
1	3	5	4	1	6	2	9	8
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6	1	2	7	6	5	3	8	4

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1	4	8	7	2	9	3	6	5
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9	6	2	8	7	3	5	1	4

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6	7	1	3	4	9	5	2	8
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3	1	9	6	2	4	7	8	5

3	7	8	6	9	2	1	5	4
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6	8	3	5	2	1	7	4	9
4	9	7	8	6	3	5	2	1
5	2	1	4	7	9	3	6	8
8	6	9	7	4	5	2	1	3
7	3	5	2	1	6	8	9	6
1	4	2	9	3	8	4	7	5

CONCEPT MAP

CIRCLES

VECTOR ALGEBRA

CONCEPT MAP

Class XI **Class XII**

CIRCLE

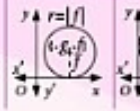
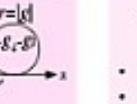
Definitions

- A circle is defined as locus of all such points in plane which remains at constant distance from a given fixed point. Fixed point is called centre and the fixed distance (distance from centre to each point of locus) is called radius of the circle.
- The second degree equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents the equation of circle, if $a = b$, $h = 0$ and

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0 \text{ and } g^2 + f^2 - ac > 0$$

Equations

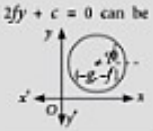
- General equation of circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $(C) \equiv (-g, -f)$ and radius $(r) = \sqrt{g^2 + f^2 - c}$
- $(x - \alpha)^2 + (y - \beta)^2 = a^2$ with centre $(C) = (\alpha, \beta)$ and radius $(r) = a$

Simplest	Touching x-axis	Touching y-axis	Touching both x & y-axis
			

Different Forms

- **Diametric Form:** $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, where (x_1, y_1) and (x_2, y_2) are end points of diameter.
- **Parametric Form:** $x^2 + y^2 + 2gx + 2fy + c = 0$ can be represented as

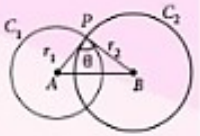
$$x = -g + r\cos\theta$$
$$y = -f + r\sin\theta$$
$$r = \sqrt{g^2 + f^2 - c}$$



Angle of intersection between two circles

If the two circles C_1 and C_2 given by $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ respectively intersect at a point P and PA, PB are the tangents at P, then the angle between the tangents at P (*i.e.* $\angle BPA$) is the angle of intersection of the two circles given by

$$\cos\theta = \frac{r_1^2 + r_2^2 - (AB)^2}{2r_1r_2}$$



Important Results

- Let $S = x^2 + y^2 + 2gx + 2fy + c$

  $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ and $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$
  - Equation of chord of the circle $S = 0$ whose mid-point is (x_1, y_1) is $T = S_1$
  - Length of intercept made by the circle $S = 0$ with x-axis $= 2\sqrt{g^2 - c}$.
  - Equation of tangent to the circle $S = 0$ is given by $T = 0$.
  - Length of intercept made by the circle $S = 0$ with y-axis $= 2\sqrt{f^2 - c}$.
  - Length of tangent from a point (x_1, y_1) to the circle $S = 0$ is given by $\sqrt{S_1}$.
  - The equation of radical axis of two circles $S_1 = 0$, $S_2 = 0$ in which the coefficient of x^2, y^2 are same is the equation of the line $S_1 - S_2 = 0$.
- Line $y = mx + c$ will touch the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.
- Equation of director circle of the circle $x^2 + y^2 = a^2$ is the circle whose radius is $\sqrt{2}$ times the radius of the original circle.

Orthogonal Circles

Two Circles are orthogonal if $\theta = 90°$

i.e., $r_1^2 + r_2^2 = (AB)^2 = 0$

or $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

VECTOR

A quantity that has magnitude as well as direction.

Multiplication of Vectors

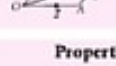
Vector (or cross) Product

Vector product of vectors is given by $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|(\sin\theta)\hat{n}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is the unit vector normal to the plane determined by $\vec{a}$ and $\vec{b}$.

Addition of Two Vectors

- **Triangle Law :** In ΔOAB, let position vectors of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ be $\vec{a}$ and $\vec{b}$ respectively. Then, $\overrightarrow{OB} = \vec{a} + \vec{b}$

- **Parallelogram Law :** In parallelogram $OABC$, let position vectors of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ be $\vec{a}$ and $\vec{b}$ respectively. Then, $\overrightarrow{OB} = \vec{a} + \vec{b}$



Properties of Vector Product

- $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$ • $\vec{a} \times \vec{a} = 0$
- $\vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b}$
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$, $\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$, $\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$, $\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- $(m\vec{a}) \times (n\vec{b}) = (mn)\,\vec{a} \times \vec{b}$
- $\vec{a} \times (\vec{b} \pm \vec{c}) = \vec{a} \times \vec{b} \pm \vec{a} \times \vec{c}$
- $(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$
- If $\vec{a}$ and $\vec{b}$ are adjacent sides of a triangle, then area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$

Projection

Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

Scalar (or Dot) Product

The scalar product of $\vec{a}$ and $\vec{b}$ written as $\vec{a} \cdot \vec{b}$ and is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$.

Properties of Scalar Product

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ • $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ • $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$,
- $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then, $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- $\vec{a} \cdot (\alpha\vec{b}) = \alpha(\vec{a} \cdot \vec{b})$
- $\vec{a} \cdot (\vec{b} \pm \vec{c}) = \vec{a} \cdot \vec{b} \pm \vec{a} \cdot \vec{c}$

Scalar Triple Product

Let $\vec{a}, \vec{b}$ are inclined at angle θ and $\vec{c}$ is inclined at angle ϕ with the vector $\vec{a} \times \vec{b}$.

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a}||\vec{b}|(\sin\theta)\hat{n} \cdot \vec{c} = |\vec{a}||\vec{b}||\vec{c}| \sin\theta \cos\phi$$

It is the volume of the parallelopiped with concurrent edges $\vec{a}, \vec{b}, \vec{c}$ and is denoted by the box product $[\vec{a}\,\vec{b}\,\vec{c}]$.

Components of a Vector

Let O be the origin and let $P(x, y, z)$ be any point in space then, $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$. x, y, z are scalar components of $\overrightarrow{OP}$ and $x\hat{i}$, $y\hat{j}$, $z\hat{k}$ are vector components of $\overrightarrow{OP}$.

Also, $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

Coplanarity

Three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar iff $\vec{a} \cdot (\vec{b} \times \vec{c})$ or $[abc] = 0$.

Properties of Scalar Triple Product

- $[\alpha\vec{a}\ \beta\vec{b}\ \gamma\vec{c}] = \alpha\beta\gamma[\vec{a}\,\vec{b}\,\vec{c}]$
- $[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$
- $[\vec{a} \times \vec{b}\ \vec{b} \times \vec{c}\ \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$
- $[\vec{a}\,\vec{b}\,\vec{c}][\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} \vec{a} \cdot \vec{u} & \vec{a} \cdot \vec{v} & \vec{a} \cdot \vec{w} \\ \vec{b} \cdot \vec{u} & \vec{b} \cdot \vec{v} & \vec{b} \cdot \vec{w} \\ \vec{c} \cdot \vec{u} & \vec{c} \cdot \vec{v} & \vec{c} \cdot \vec{w} \end{vmatrix}$
- $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$ • $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ if any two of the vectors are parallel
- $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ if $\vec{a}, \vec{b}, \vec{c}$ are coplanar or $\vec{c} = \alpha\vec{a} + \beta\vec{b}$
- $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
- $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Series 8

Statistics and Probability

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part – A:

(1) It consists of two sections-I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

(1) It consists of three sections-III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION-I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the range of the data 35, 50, 48, 62, 27, 39, 43, 72, 56, 68.

2. Find the range of the following set of observations
2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3.

OR

Find the mean deviation about the mean for the following data :
6, 7, 10, 12, 13, 4, 8, 12

3. Find the mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean.

4. If the range of the given data arranged in descending order is 148, then find the value of x.
x, 178, 146, 108, 105, 100, 100, 98, 96

5. Let the standard deviation of a distribution is 30. If each item is raised by 3, then find new S.D.

6. The mean and variance of n observations $x_1, x_2, x_3, \ldots, x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400$, then find the value of n.

OR

A set of n values $x_1, x_2, \ldots, x_n$ has standard deviation σ. Find the standard deviation of n values $x_1 + k, x_2 + k, \ldots, x_n + k$.

7. Variance of the data, 2, 4, 5, 6, 8, 17 is 23.33. Then find the variance of 4, 8, 10, 12, 16, 34.

8. If the standard deviation of 3, 8, 6, 10, 12, 9, 11, 10, 12, 7 is 2.71, then find the standard deviation of 30, 80, 60, 100, 120, 90, 110, 100, 120, 70.

OR

The mean and variance of n values of a variable x are 0 and σ^2, respectively. If the variable $y = x^2$, then find the mean of y.

9. A die is thrown repeatedly until a six comes up. Write the sample space for this experiment.

10. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 yellow and 3 red balls. If it shows head, we throw a die. Write the sample space for this experiment.

11. Find the total number of elementary events associated to the random experiment of throwing three die together.

12. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Write the possible outcomes of this experiment.

OR

Two coins (a ₹ 2 coin and a ₹ 5 coin) are tossed once. Find the total number of elements in sample space.

13. Let the sample space associated with an experiment is $S = \{1, 2, 3, 4, 5, 6\}$ and an event is $E = \{1, 3, 5\}$, then find E' or $\bar{E}$.

14. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

15. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

16. A card is drawn from a well shuffled deck of 52 cards, then find the probability of red king card.

OR

In a single throw of a die, find the probability of getting an even prime number.

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. In a particular school, the following is the record of goals scored by team A in a football match.

Number of goals scored	0	1	2	3	4
Number of matches	1	9	7	5	3

Based on the above information, answer the following questions.

(i) The mean number of goals scored per match by team A is

(a) 2 (b) 3 (c) 4 (d) 5

(ii) The standard deviation of the goals scored, for the team A, is

(a) 2.35 (b) 1.06 (c) 1.095 (d) 2.009

(iii) Coefficient of variation of the goals scored, for the team A, is

(a) 64.75 (b) 54.75
(c) 45.57 (d) None of these

(iv) For team B, if mean number of goals scored per match was 1 with standard deviation 1.25 goals, then find the coefficient of variation of the goals scored by team B.

(a) 72.5 (b) 63.5 (c) 95.8 (d) 125

(v) Which team may be considered more consistent if data for team B is taken from part (iv).

(a) Team A
(b) Team B
(c) Both team are performing on same level
(d) Can't be compared

18. Arjun gave his sister a box containing 6 red, 4 white and 5 black balls. She draws 4 balls from the box at random.

Based on the above information, answer the following questions.

(i) If S be the sample space associated with the random experiment, then $n(S)$ =
(a) 1300 (b) 1365 (c) 1470 (d) 1465

(ii) If A be the event of getting 1 red, 1 white and 2 back balls, then $P(A)$ is equal to
(a) 240/1365 (b) 1/1365
(c) 4/1365 (d) 80/1365

(iii) If B be the event of getting 2 red, 1 white and 1 black balls, then $P(B)$ is equal to
(a) 5/1370 (b) 1/1470
(c) 180/1365 (d) 300/1365

(iv) If C be the event of getting 1 red, 2 white and 1 black balls, then $P(C)$ is equal to
(a) 180/1365 (b) 1/1365
(c) 240/1370 (d) 1/1465

(v) Find the probability that among the balls drawn, there is atleast one ball of each colour.
(a) 1/1365 (b) 48/9170
(c) 48/91 (d) 720/1475

PART-B

SECTION-III

All questions are compulsory. In case of internal choices attempt any one.

19. Find the standard deviation of first n odd natural numbers.

20. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. Find the variance of the combined data set.

OR

Find the mean of first 10 multiples of 3.

21. Find the standard deviation of 12, 23, 34, 45, 56, 67 and 78.

22. Find the standard deviation of the first $(2n + 1)$ natural numbers.

23. Find the standard deviation for the following data :

x_i	3	8	13	18	23
f_i	7	10	15	10	6

OR

Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that
(i) A is a simple event
(ii) B and C are compound events
(iii) A and B are mutually exclusive events.

24. Find the probability of getting the sum as a prime number when two dice are thrown together.

25. The scores of batsman A in 10 different test matches were 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the standard deviation.

26. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

27. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows an even number". Check whether, E and F are mutually exclusive or not.

OR

A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Find the total number of possible outcomes for this experiment.

28. What is the probability that all L's come together in the word PARALLEL?

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. Find the mean deviation about the mean for the following data :
12, 3, 18, 17, 4, 9, 17, 19, 20, 15

30. The following data gives the weights (in grams) of 15 oranges picked up from a basket:
106, 107, 76, 82, 109, 107, 115, 92, 127, 95, 123, 126, 76, 92, 86
Calculate the mean deviation about the median.

31. Find the variance of first 10 multiples of 3.

OR

The marks obtained by 10 students in an examination are as follows :
40, 62, 58, 73, 86, 83, 50, 54, 71, 75
Calculate the standard deviation.

32. Let $\bar{x}$ and $\bar{y}$ are the A.M.'s of the variable x, y respectively and σ_x, σ_y are the standard deviations of the variables x and y respectively and a, b are

constants. If the variables x and y are such that $y = ax + b$, prove that

(i) $\bar{y} = a\bar{x} + b$ (ii) $\sigma_y = |a|\sigma_x$

33. A coin is tossed three times, consider the following events :

A : 'No head appears'

B : 'Exactly one head appears'

C : 'At least two heads appear'

Do they form a set of mutually exclusive and exhaustive events?

34. A bag contains 9 balls of which 4 are red, 3 are blue and 2 are yellow. The balls are similar in shape and size. A ball is drawn at random from the bag. Calculate the probability that it will be:

(i) red (ii) not blue

(iii) either red or blue

OR

What is the probability that:

(i) a non-leap year has 53 Tuesdays?

(ii) a leap year has 53 Wednesdays?

(iii) a leap year has 53 Fridays and 53 Saturdays?

35. If $\bar{x}$ is the mean and mean deviation from mean is M.D. $(\bar{x})$, then find the number of observations lying between $\bar{x}$ – M.D. $(\bar{x})$ and $\bar{x}$ + M.D.$(\bar{x})$ for the following data : 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

SECTION - V

All questions are compulsory. In case of internal choices attempt any one.

36. Calculate the mean deviation about the mean of the set of first n natural numbers when n is even natural number.

OR

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases :

(i) If the wrong item is omitted.

(ii) If it is replaced by 12.

37. Find the S.D. of the following distribution :

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

OR

20 cards are numbered from 1 to 20. One card is then drawn at random. What is the probability that the number on the card drawn is

(i) a prime number? (ii) an odd number?

(iii) a multiple of 5? (iv) not divisible by 3?

38. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events :

A : the sum is greater than 8

B : 2 occurs on either die

C : the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

OR

A die is thrown. Describe the following events :

(i) A : a number less than 7

(ii) B : a number greater than 7

(iii) C : a multiple of 3

(iv) D : a number less than 4

(v) E : an even number greater than 4

(vi) F : a number not less than 3.

Also find $A \cup B$, $A \cap B$, $E \cup F$, $D \cap E$, $A - C$, $D - E$, F', $E \cap F'$.

SOLUTIONS

1. Range = 72 – 27 = 45.

2. Range = 9 – 2 = 7

OR

Mean of the given data is

$$\bar{x} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$$

The values of the deviations, *i.e.*, $|x_i - \bar{x}|$ are 3, 2, 1, 3, 4, 5, 1, 3

$$\therefore \quad \text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{8}|x_i - \bar{x}|}{8}$$

$$= \frac{3+2+1+3+4+5+1+3}{8} = \frac{22}{8} = 2.75$$

3. Mean, $\bar{x} = \dfrac{2+9+9+3+6+9+4}{7} = \dfrac{42}{7} = 6$

The values of deviations from mean *i.e.*, $|x_i - \bar{x}|$ are 4, 3, 3, 3, 0, 3, 2

$$\therefore \quad \text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{7}|x_i - \bar{x}|}{7} = \frac{4+3+3+3+0+3+2}{7} = 2.57$$

4. Range = Maximum value – least value

$\Rightarrow$ $148 = x - 96 \Rightarrow x = 148 + 96 = 244$

5. S.D. of a series unaltered if each item is raised (reduced) by the some scalar quantity, *i.e.*, S.D. is independent of change of origin.

$\therefore$ S.D. = 30

6. We know, Variance $= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = \sigma^2$

$\Rightarrow \quad 0 = \frac{1}{n}(400) - (5)^2 \Rightarrow \frac{400}{n} = 25 \Rightarrow n = 16$

OR

If each observation is increased by a constant k, then standard deviation is unchanged.

$\therefore$ Required standard deviation $= \sigma$

7. When each observation is multiplied by 2, then variance is also multiplied by 2^2.

$\therefore$ New Variance $= 2^2 \times 23.33 = 93.32$

8. Standard deviation $= \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

If each term is multiplied by 10, then

S.D. $= \sqrt{\frac{1}{n}\Sigma(10x_i - 10\bar{x})^2}$

$\Rightarrow$ S.D. $= \sqrt{\frac{1}{n} \times 100 \times \Sigma(x_i - \bar{x})^2} = 10 \times \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$

Hence, new S.D. $= 10 \times 2.71 = 27.1$

OR

Given that, Mean $= 0$ and variance $= \sigma^2$ (for x)

Therefore, variance $= \sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\Rightarrow \quad \sigma^2 = \frac{\Sigma x^2}{n} - 0 \Rightarrow \sigma^2 = \frac{\Sigma x^2}{n}$

Now, $\quad y = x^2$

$\Rightarrow \quad \Sigma y = \Sigma x^2 \quad \therefore \quad \frac{\Sigma y}{n} = \frac{\Sigma x^2}{n} = \sigma^2$

9. The sample space is

$S = \{6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), ...\}$

It can be observed that infinite number of possibilities occur.

10. Let H and T represent a head and a tail of a coin, respectively.

Also, let red balls are represented by R_1, R_2 and R_3 and yellow balls are represented by Y_1 and Y_2. Then, the sample space,

$S = \{TR_1, TR_2, TR_3, TY_1, TY_2, H1, H2, H3, H4, H5, H6\}$

11. When three dice are tossed together, then the total number of possible outcomes $= 6^3 = 6 \times 6 \times 6 = 216$

12. The sample space for this experiment is

$S = \{RR, RB, BR, BB\}$, where R denotes the red ball and B denotes the black ball.

OR

The sample space is $S = \{HH, HT, TH, TT\}$

Thus, $n(S) = 4$

13. Given that, $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$

$\therefore \quad \bar{E} = S - E = \{2, 4, 6\}$.

14. The sample space S for selecting three bulbs at random from a lot is given by

$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}$

where D indicates a defective bulb and N a non-defective bulb.

15. The sample space S for the given experiment is

$S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$.

16. The outcomes in the sample space S are 52.

$\therefore \quad n(S) = 52$

Let E be the event of getting a red king card

$\because$ Out of 4 kings, 2 are red and 2 are black

$\therefore \quad n(E) = 2$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

OR

Let $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space and $E = \{2\}$ be event of getting an even prime number.

$\Rightarrow \quad n(S) = 6$ & $n(E) = 1$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{1}{6}$

17. We have the following table for the given data :

Number of goals (x_i)	Number of matches (f_i)	x_i^2	$f_i x_i$	$f_i x_i^2$
0	1	0	0	0
1	9	1	9	9
2	7	4	14	28
3	5	9	15	45
4	3	16	12	48
Total	25		50	130

Here, $N = \Sigma f_i = 25$; $\Sigma f_i x_i = 50$ and $\Sigma f_i x_i^2 = 130$

(i) (a) : For team A, Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{50}{25} = 2$

(ii) (c) : For team A, standard deviation

$\sigma = \frac{1}{N}\sqrt{N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2} = \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$

$= \frac{5}{25}\sqrt{130 - 100} = \frac{\sqrt{30}}{5} = \frac{5.477}{5} = 1.095$

(iii) (b) : For team A, coefficient of variation

$$= \frac{\sigma}{\bar{x}} \times 100 = \frac{1.095}{2} \times 100 = 54.75$$

(iv) (d) : For team B, coefficient of variation

$$= \frac{\sigma}{\bar{x}} \times 100 = \frac{1.25}{1} \times 100 = 125$$

(v) (a) : Since, coefficient of variation of goals of team A is less than that of team B.

$\therefore$ Team A is more consistent than team B.

18. (i) (b) : Total number of balls = 15

$$\therefore\ n(S) = {}^{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365$$

(ii) (a) : $n(A) = {}^6C_1 \times {}^4C_1 \times {}^5C_2 = 6 \times 4 \times 10 = 240$

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{240}{1365}$$

(iii) (d) : $n(B) = {}^6C_2 \times {}^4C_1 \times {}^5C_1 = 15 \times 4 \times 5 = 300$

$$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{300}{1365}$$

(iv) (a) : $n(C) = {}^6C_1 \times {}^4C_2 \times {}^5C_1 = 6 \times 6 \times 5 = 180$

$$\therefore\ P(C) = \frac{n(C)}{n(S)} = \frac{180}{1365}$$

(v) (b) : P(getting atleast one ball of each colour)

$= P(A \cup B \cup C) = P(A) + P(B) + P(C)$

[$\because A, B, C$ are mutually exclusive events]

$$= \frac{240}{1365} + \frac{300}{1365} + \frac{180}{1365} = \frac{720}{1365} = \frac{48}{91}$$

19. Mean of first n odd natural numbers

$$\bar{x} = \frac{1+3+5+....+(2n-1)}{n} = \frac{n^2}{n} = n$$

Sum of square of first n odd natural numbers *i.e.*,

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2$$
$$= (1^2 + 2^2 + ... + (2n)^2) - (2^2 + 4^2 + ... + (2n)^2)$$
$$= \frac{(2n)(2n+1)(2(2n)+1)}{6} - 2^2(1^2 + 2^2 + ... + n^2)$$
$$= \frac{n(2n+1)(2n-1)}{3}$$

$\therefore$ Standard deviation $= \sqrt{\frac{n(2n+1)(2n-1)}{3n} - n^2}$

$$= \sqrt{\frac{n^2-1}{3}}$$

20. We have, $\sigma_1^2 = 4, \bar{x} = 2,\ \sigma_2^2 = 5, \bar{y} = 4$

Now, $\frac{\sum x_i}{5} = 2 \Rightarrow \sum x_i = 10$

Similarly, $\sum y_i = 20$

$$\sigma_1^2 = \left(\frac{1}{5}\sum x_i^2\right) - \bar{x}^2 \Rightarrow 4 = \frac{1}{5}\sum x_i^2 - 4 \Rightarrow \sum x_i^2 = 40$$

$$\sigma_2^2 = \left(\frac{1}{5}\sum y_i^2\right) - \bar{y}^2 \Rightarrow 5 = \frac{1}{5}\sum y_i^2 - 16$$

$$\Rightarrow \sum y_i^2 = 105$$

Now, combined variance is given by

$$\sigma^2 = \frac{1}{10}\left(\sum x_i^2 + \sum y_i^2\right) - \left(\frac{\bar{x}+\bar{y}}{2}\right)^2$$
$$= \frac{1}{10}(40+105) - 9 = \frac{145-90}{10} = \frac{55}{10} = \frac{11}{2}.$$

OR

Here, x_i are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

$\therefore\ \Sigma x_i = 3+6+9+12+15+18+21+24+27+30 = 165$

and $n = 10$

Now, Mean $(\bar{x}) = \frac{165}{10} = 16.5$

21. Standard deviation of ungrouped data is,

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

Here, $\bar{x} = \frac{12+23+34+45+56+67+78}{7} = 45$

Now, $\sum_{i=1}^{7}(x_i - \bar{x})^2 = (12-45)^2 + (23-45)^2 + (34-45)^2$

$+ (45-45)^2 + (56-45)^2 + (67-45)^2 + (78-45)^2$

$= 1089 + 484 + 121 + 0 + 121 + 484 + 1089 = 3388$

$$\therefore\ \sigma = \sqrt{\frac{3388}{7}} = \sqrt{484} = 22$$

22. Here, $\bar{x}$ = Mean $= \frac{1+2+3+...+(2n+1)}{(2n+1)}$

$$= \frac{(2n+1)(2n+2)}{2(2n+1)} = n+1$$

Variance $= \sigma^2 = \frac{1}{2n+1}\sum (x_i - \bar{x})^2$

$$= \frac{2}{2n+1}[n^2 + (n-1)^2 + ... + (1)^2]$$

$$= \frac{2}{2n+1} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{3}$$

$$\Rightarrow \text{S.D.} = \sqrt{\text{Variance}} = \sqrt{\frac{n(n+1)}{3}}.$$

23. Let us form the following table :

x_i	f_i	f_ix_i	x_i^2	$f_ix_i^2$
3	7	21	9	63
8	10	80	64	640
13	15	195	169	2535
18	10	180	324	3240
23	6	138	529	3174
Total	48	614		9652

Now, $\sigma = \frac{1}{N}\sqrt{N\Sigma f_ix_i^2 - (\Sigma f_ix_i)^2} = \frac{1}{48}\sqrt{48\times 9652 - (614)^2}$

$= \frac{1}{48}\sqrt{463296 - 376996} = \frac{1}{48} \times 293.77 = 6.12$

OR

We have, $A = \{(1, 1)\}$, $B = \{(1, 2), (2, 1)\}$
and $C = \{(1, 3), (3, 1), (2, 2)\}$
(i) Since A consists of a single sample point, it is a simple event.
(ii) Since both B and C contain more than one sample point, therefore each one of them is a compound event.
(iii) Since $A \cap B = \phi$.
$\therefore$ A and B are mutually exclusive events.

24. Total number of possible outcomes = 36
$\because$ The sum should be prime number *i.e.*, 2, 3, 5, 7, 11.
Sum $2 \equiv (1, 1)$
Sum $3 \equiv (1, 2), (2, 1)$
Sum $5 \equiv (1, 4), (4, 1), (2, 3), (3, 2)$
Sum $7 \equiv (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)$
Sum $11 \equiv (5, 6), (6, 5)$
$\therefore$ Number of favourable outcomes = 15
Hence, required probability $= \frac{15}{36} = \frac{5}{12}$

25. To find the standard deviation, we construct the following table, taking assumed mean $a = 48$.

x_i	$d_i = x_i - a$	d_i^2
34	−14	196
38	−10	100
42	−6	36
44	−4	16
46	−2	4
48	0	0
54	6	36
55	7	49
63	15	225
70	22	484
Total	14	1146

Now, $\sigma(\text{S.D.}) = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{1146}{10} - \left(\frac{14}{10}\right)^2}$

$= \sqrt{114.6 - 1.96} = \sqrt{112.64} = 10.61$ nearly.

26. Six employees can be seated in row in six desks in 6! ways. Married couple can occupy adjacent seats in the following 5 ways.
1 – 2, 2 – 3, 3 – 4, 4 – 5, 5 – 6
Also, they can interchange their seats and the remaining 4 seats can be occupied by remaining 4 employees in 4! ways.
$\therefore$ Number of ways in which married couple will have adjacent seats $= 5 \times 2! \times 4!$
So, number of ways in which married couple will have non-adjacent seats $= 6! - 5 \times 2! \times 4! = 480$
Hence, required probability $= \frac{480}{720} = \frac{2}{3}$

27. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.
Now, E = The event "die shows 4" $= \{4\}$
and F = The event "die shows an even number"
$= \{2, 4, 6\}$
$\because$ $E \cap F = \{4\} \neq \phi$
$\therefore$ E and F are not mutually exclusive.

OR

Corresponding to each doublet we get two outcomes.
For example, corresponding to (1, 1) we get (1, 1, H), (1, 1, T).
Thus, total number of possible outcomes $= 36 + 6 = 42$.

28. Total number of possible outcomes $= \frac{8!}{3!2!}$
Total number of favourable outcomes when (3*L*'s consider as one letter) $= \frac{6!}{2!}$
Hence, required probability $= \frac{6!}{8!/3!} = \frac{3}{28}$

29. Let the mean of the given data be $\bar{x}$. Then,
$\bar{x} = \frac{12+3+18+17+4+9+17+19+20+15}{10}$
$= \frac{134}{10} = 13.4$

For M.D. (about mean), prepare the following table:

x_i	$\lvert d_i \rvert = \lvert x_i - \bar{x} \rvert = \lvert x_i - 13.4 \rvert$
12	1.4
3	10.4
18	4.6
17	3.6

4	9.4
9	4.4
17	3.6
19	5.6
20	6.6
15	1.6
Total	51.2

$\therefore$ Mean Deviation about mean,

$$\text{M.D. } (\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} = \frac{1}{n}\sum_{i=1}^{n} |d_i| = \frac{51.2}{10} = 5.12$$

Hence, Mean Deviation about the mean, M.D. $(\bar{x}) = 5.12$

30. Arranging the weights of the oranges in ascending order, we get

76, 76, 82, 86, 92, 92, 95, 106, 107, 107, 109, 115, 123, 126, 127

Here, $n = 15$, which is odd

$\therefore$ M = Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ observation

$= 8^{\text{th}}$ observation $= 106$

Weight (in gms)	$\lvert x_i - M \rvert$
76	30
76	30
82	24
86	20
92	14
92	14
95	11
106	0
107	1
107	1
109	3
115	9
123	17
126	20
127	21
Total	215

$\therefore$ Mean deviation about median $= \frac{215}{15} = 14.3$

31. Variance, $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$$= \left\{\frac{3^2 + 6^2 + 9^2 + \ldots.. + 30^2}{10} - \frac{3(1+2+\ldots..+10)}{10}\right\}$$

$$= \frac{3^2 \times \left\{1^2 + 2^2 + 3^2 + \ldots + 10^2\right\}}{10} - \frac{3\times10\times(10+1)}{2\times10}$$

$$= \frac{9 \times 10 \times (10+1)(2\times10+1)}{6\times10} - (16.5)^2$$

$$= \left(\frac{9\times10\times11\times21}{6\times10} - 272.25\right) = (346.5 - 272.25) = 74.25.$$

$\therefore$ Variance, $\sigma^2 = 74.25$.

OR

Let the assumed mean be 62.

x_i	$d_i = x_i - 62$	d_i^2
40	−22	484
50	−12	144
54	−8	64
58	−4	16
62	0	0
71	9	81
73	11	121
75	13	169
83	21	441
86	24	576
Total	$\Sigma d_i = 32$	$\Sigma d_i^2 = 2096$

Now, $\sigma^2 = \frac{1}{n}\Sigma d_i^2 - \left(\frac{\Sigma d_i}{n}\right)^2$

$\Rightarrow \sigma = \sqrt{\frac{2096}{10} - \left(\frac{32}{10}\right)^2} = \sqrt{209.6 - 10.24} = \sqrt{199.36}$

$\Rightarrow \sigma = 14.12$

Thus, the standard deviation is 14.12.

32. Let number of observations for variable x be n.

Then for each value of x, y will have one value.

$\therefore$ Number of observations for variable y will be n.

(i) Now, $\bar{y} = \frac{\Sigma y}{n} = \frac{\Sigma(ax+b)}{n}$ $[\because\ y = ax + b]$

$$= \frac{\Sigma ax + \Sigma b}{n} = \frac{a\Sigma x + bn}{n} = \frac{a\Sigma x}{n} + b = a\bar{x} + b$$

(ii) $\sigma_x^2 = \frac{\Sigma(x-\bar{x})^2}{n}$ ...(i)

$$\sigma_y^2 = \frac{\Sigma(y-\bar{y})^2}{n} = \frac{\Sigma[ax+b-(a\bar{x}+b)]^2}{n} \quad [\because\ \bar{y} = a\bar{x} + b]$$

$$= \frac{\Sigma[a(x-\bar{x})]^2}{n} = a^2\frac{\Sigma(x-\bar{x})^2}{n} = a^2\sigma_x^2 \quad \text{[From (i)]}$$

$\therefore$ $\sigma_y = |a|\sigma_x$ $[\because\ \sigma_x, \sigma_y > 0]$

33. If S be the sample space of tossing a coin three times then S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Now, A (Event of getting no head) = {TTT}

B (Event of getting exactly one head) = {HTT, THT, TTH}

C (Event of getting atleast two heads) = {HHT, HHH, HTH, THH}

Clearly, $A \cap B = \phi$, $B \cap C = \phi$ and $A \cap C = \phi$

So, A, B and C are mutually exclusive events.

Also, $A \cup B \cup C = S$

So, A, B and C are exhaustive events.

34. (i) Total number of possible outcomes = 9

Number of red balls = 4

$\therefore$ Number of favourable outcomes = 4

Hence, required probability = 4/9

(ii) Number of balls which are not blue = 4 + 2 = 6

$\therefore$ Number of favourable outcomes = 6

Hence, required probability = 6/9 = 2/3

(iii) Number of balls which are either red or blue = 4 + 3 = 7

$\therefore$ Number of favourable outcomes = 7

Hence, required probability = 7/9

OR

Refer to Answer 12, page no. 356 of MTG CBSE Champion Mathematics, Class 11.

35. Given data arranged in ascending order is 30, 34, 38, 40, 42, 44, 50, 51, 60, 66.

$$\bar{x} = \frac{30+34+38+40+42+44+50+51+60+66}{10}$$

$$= \frac{455}{10} = 45.5$$

Now, $|x_i - \bar{x}| = 15.5, 11.5, 7.5, 5.5, 3.5, 1.5, 4.5, 5.5, 14.5, 20.5$

Mean deviation from the mean

$$= \text{M.D.}(\bar{x}) = \frac{\Sigma |x_i - \bar{x}|}{10} = \frac{1}{10}(90) = 9$$

Now, $\bar{x} - \text{M.D.}(\bar{x}) = 45.5 - 9 = 36.5$

and $\bar{x} + \text{M.D.}(\bar{x}) = 45.5 + 9 = 54.5$

Given observation which lie between 36.5 and 54.5 are 38, 40, 42, 44, 50, 51 which are six in number.

$\therefore$ Six observations of given data lie between $\bar{x} - \text{M.D.}(\bar{x})$ and $\bar{x} + \text{M.D.}(\bar{x})$.

36. *Refer to Answer 32, page no. 328 of MTG CBSE Champion Mathematics, Class 11.*

OR

Refer to Answer 60, page no. 334 of MTG CBSE Champion Mathematics, Class 11.

37. Let us prepare the following table after taking assumed mean $A = 34.5$ and $h = 10$.

x_i	f_i	$u_i = \frac{x_i - 34.5}{10}$	u_i^2	$f_i u_i$	$f_i u_i^2$
4.5	1	−3	9	−3	9
14.5	5	−2	4	−10	20
24.5	12	−1	1	−12	12
34.5	22	0	0	0	0
44.5	17	1	1	17	17
54.5	9	2	4	18	36
64.5	4	3	9	12	36
Total	70			22	130

Now, $\text{S.D.} = \sigma = h\sqrt{\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2}$

$$= 10\sqrt{\frac{130}{70} - \left(\frac{22}{70}\right)^2} = 10\sqrt{\frac{130\times 70 - 484}{70\times 70}}$$

$$= \frac{10}{70}\sqrt{9100 - 484} = \frac{1}{7}\sqrt{8616} = \frac{92.82}{7} = 13.26$$

OR

Refer to Answer 25, page no. 357 of MTG CBSE Champion Mathematics, Class 11.

38. When two dice are thrown, there are $6 \times 6 = 36$ possible outcomes.

$\therefore$ S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ A = The sum is greater than 8

= {(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

B = 2 occurs on either die

= {(1, 2), (2, 1), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (3, 2), (4, 2), (5, 2), (6, 2)}

C = The sum is at least 7 and a multiple of 3

= {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

It can be seen that $A \cap B = \phi$, $B \cap C = \phi$

and $A \cap C$ = {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

(i) Since $A \cap B = \phi$. So A and B are mutually exclusive.

(ii) Since $B \cap C = \phi$. So B and C are mutually exclusive.

(iii) Since $A \cap C \neq \phi$. So A and C are not mutually exclusive.

OR

Refer to Answer 27, page no. 358 of MTG CBSE Champion Mathematics, Class 11.

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 | **Time Taken : 60 Min.**

Series 8 : Straight Lines

Only One Option Correct Type

1. If the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is rotated about the origin through 90°, then their equations in the new position are given by
   (a) $ax^2 - 2hxy + by^2 = 0$
   (b) $ax^2 - 2hxy - by^2 = 0$
   (c) $bx^2 - 2hxy + ay^2 = 0$
   (d) $bx^2 + 2hxy + ay^2 = 0$

2. The co-ordinate axes are rotated about the origin O in the counter-clockwise direction through an angle 60°. If p and q are the intercepts made on the new axes by a straight line whose equation referred to the original axes is $x + y = 1$, then $\frac{1}{p^2} + \frac{1}{q^2} =$
   (a) 2 (b) 4 (c) 6 (d) 8

3. If the equation of the pair of straight lines passing through (1, 1), one making an angle θ with the positive direction of x-axis and the other making the same angle with the positive direction of y-axis is $x^2 - (a+2)xy + y^2 + a(x + y - 1) = 0, a \neq -2$, then $\sin 2\theta =$
   (a) $a - 2$ (b) $a + 2$ (c) $\frac{2}{a+2}$ (d) $\frac{2}{a}$

4. Locus of the centroid of the triangle whose vertices are ($a \cos t$, $a \sin t$), ($b \sin t$, $-b \cos t$) and (1, 0), where t is a parameter, is
   (a) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$
   (b) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$
   (c) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
   (d) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

5. If the gradient of one of the lines given by $x^2 + hxy + 2y^2 = 0$ is twice that of the other, then $h =$
   (a) ± 2 (b) ± 3 (c) ± 1 (d) $\pm 3/2$

6. The line $(k + 1)^2x + ky - 2k^2 - 2 = 0$ passes through a fixed point. Distance of this point from (–2,–1) is
   (a) 3 (b) 4 (c) 5 (d) 6

One or More Than One Option(s) Correct Type

7. The lines $x + y - 1 = 0$, $(m - 1)x + (m^2 - 7)y - 5 = 0$, and $(m - 2)x + (2m - 5)y = 0$ are
   (a) concurrent for three values of m
   (b) concurrent for no value of m
   (c) parallel for one value of m
   (d) parallel for two values of m

8. If $x^2 + 2hxy + y^2 = 0$ represents the equation of the straight lines through the origin which make an angle α with the straight line $y + x = 0$, then
   (a) $\sec 2\alpha = h$ (b) $\cos\alpha = \sqrt{(1+h)/(2h)}$
   (c) $2\sin\alpha = \sqrt{(1+h)/h}$ (d) $\cot\alpha = \sqrt{(h+1)/(h-1)}$

9. The equation of the diagonal of the square formed by the pairs of lines $xy + 4x - 3y - 12 = 0$ and $xy - 3x + 4y - 12 = 0$ is
   (a) $x - y = 0$ (b) $x + y + 1 = 0$
   (c) $x + y = 0$ (d) $x - y + 1 = 0$

10. If graph of $xy = 1$ is reflected in $y = 2x$ to give the graph $12x^2 + rxy + sy^2 + t = 0$, then
    (a) $r = 7$ (b) $s = -12$
    (c) $t = 25$ (d) $r + s = -19$

11. A straight line passing through the point $A(-2, -3)$ cuts lines $x + 3y = 9$ and $x + y + 1 = 0$ at B and C, respectively. If $AB \cdot AC = 20$, then equation of the possible line is
    (a) $x - y = 1$ (b) $x - y + 1 = 0$
    (c) $3x - y + 3 = 0$ (d) $3x - y = 3$

12. Let $0 < p < q$ and $a \neq 0$ such that the equation $px^2 + 4\lambda xy + qy^2 + 4a(x + y + 1) = 0$ represents a pair of straight lines, then a can lie in the interval
(a) $(-\infty, \infty)$ (b) $(-\infty, p]$
(c) $[p, q]$ (d) $[q, \infty)$

13. $9x^2 + 2hxy + 4y^2 + 6x + 2fy - 3 = 0$ represents two parallel lines if
(a) $h = 6, f = 2$ (b) $h = -6, f = -2$
(c) $h = -6, f = 2$ (d) $h = 6, f = -2$

Comprehension Type

Through a given point $O(0, 0)$, a straight line is drawn to cut two given straight lines $a_1x + b_1y = 1$ and $a_2x + b_2y = 1$ in L and M respectively. Then the locus of a point N on a variable line

14. When ON is the arithmetic mean of OL and OM is
(a) $2a_1a_2x^2 + 2b_1b_2y^2 + 2(a_1b_2 + a_2b_1)yx - (a_1 + a_2)x - (b_1 + b_2)y = 0$
(b) $2b_1b_2x^2 + 2a_1a_2y^2 + 2(a_1b_2 + a_2b_1)yx - (a_1 + a_2)x - (b_1 + b_2)y = 0$
(c) $2b_1b_2x^2 + 2a_1a_2y^2 - 2(a_1b_2 + a_2b_1)yx - (a_1 + a_2)x - (b_1 + b_2)y = 0$
(d) None of these

15. When ON is the geometric mean of OL and OM is
(a) $a_1a_2x^2 + b_1b_2y^2 = 1$
(b) $a_1a_2x^2 + b_1b_2y^2 + (a_1b_2 + a_2b_1)xy = 1$
(c) $a_1a_2x^2 + b_1b_2y^2 + (a_1b_2 + a_2b_1)xy = 0$
(d) none of these

Matrix Match Type

16. Match the following columns:

	Column-I		Column-II
P.	A straight line with negative slope passing through (1, 4) meets the coordinate axes at A and B. The minimum length of $OA + OB$, O being the origin, is	1.	$5\sqrt{2}$
Q.	If the point P is symmetric to the point $Q(4,-1)$ with respect to the bisector of the first quadrant, then the length of PQ is	2.	$3\sqrt{2}$
R.	On the portion of the straight line $x + y = 2$ between the axis a square is constructed away from the origin, with this portion as one of its sides. If d denotes the perpendicular distance of a side of this square from the origin then the maximum value of d is	3.	9

	P	Q	R
(a)	3	2	1
(b)	3	1	2
(c)	1	2	3
(d)	2	1	3

Numerical Value Type

17. The value of λ if $2x^2 + 7xy + 3y^2 + 8x + 14y + \lambda = 0$ represents a pair of straight line is _____.

18. A line passes through the point $P(2, 3)$ and makes an angle of 30° with the positive direction of x-axis. It meets the lines represented by $x^2 - 2xy - y^2 = 0$ at the points A and B such that $PA \cdot PB = a$, the value of $7(a - 17)^2$ is equal to _____.

19. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0) is _____.

20. Consider a ΔABC whose sides AB, BC and CA are represented by the straight lines $2x + y = 0$, $x + py = q$ and $x - y = 3$, respectively. If the point $P(2, 3)$ is the orthocenter, then the value of $(p + q)/10$ is _____.

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CONCEPT BOOSTER

Class XII

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR

Indefinite Integrals

Let $F(x)$ be a differentiable function of x such that $\frac{d}{dx}[F(x)] = f(x)$. Then $F(x)$ is called integral of $f(x)$.

Symbolically, it is written as $\int f(x)dx = F(x)$.

Here, $F(x)$ is also called anti-derivative (or primitive function) of $f(x)$.

As the differential coefficient of a constant is zero, we have

$\frac{d}{dx}(F(x)) = f(x) \Rightarrow \frac{d}{dx}[F(x)+c] = f(x)$

$\therefore \int f(x)dx = F(x)+c$, where c is called the constant of integration and can take any real value.

SOME FUNDAMENTAL INTEGRALS

- $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c, n \neq -1$
- $\int \frac{1}{x}\,dx = \log|x|+c, x \neq 0$
- $\int e^x\,dx = e^x+c$
- $\int a^x\,dx = \frac{a^x}{\log a}+c\,(a>0)$
- $\int \sin x\,dx = -\cos x+c$
- $\int \cos x\,dx = \sin x+c$
- $\int \sec^2 x\,dx = \tan x+c$
- $\int \operatorname{cosec}^2 x\,dx = -\cot x+c$
- $\int \operatorname{cosec} x \cdot \cot x\,dx = -\operatorname{cosec} x+c$
- $\int \sec x \cdot \tan x\,dx = \sec x+c$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$
- $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
- $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c$
- $\int \cot x\,dx = \log|\sin x|+c = -\log|\operatorname{cosec} x|+c$
- $\int \tan x\,dx = -\log|\cos x|+c = \log|\sec x|+c$
- $\int \sec x\,dx = \log|\sec x+\tan x|+c$
- $\int \operatorname{cosec} x\,dx = \log|(\cot x-\operatorname{cosec} x)|+c$
  $= \log\left|\tan\frac{x}{2}\right|+c$

STANDARD FORMULAE

- $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|+c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|+c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

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- $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$
- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$
- $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

METHODS OF INTEGRATION

There are three methods for finding integral of a function.

(i) Integration by substitution

If integral is of the form $\int f(g(x))g'(x)dx$, then put $g(x) = t$, provided $\int f(t)dt$ exists.

Integrals	Substitution
• $\int \frac{f'(x)}{f(x)}dx$	$f(x) = t$
• $\int \frac{f'(x)}{\sqrt{f(x)}}dx$	$f(x) = t$

Standard Substitutions

For terms of the form

- $x^2 + a^2$ or $\sqrt{x^2+a^2}$, put $x = a\tan\theta$ or $a\cot\theta$
- $x^2 - a^2$ or $\sqrt{x^2-a^2}$, put $x = a\sec\theta$ or $a\,\text{cosec}\theta$
- $a^2 - x^2$ or $\sqrt{a^2-x^2}$, put $x = a\sin\theta$ or $a\cos\theta$
- If both $\sqrt{a+x}, \sqrt{a-x}$ are present, put $x = a\cos\theta$.
- $\sqrt{(x-a)(b-x)}$, put $x = a\cos^2\theta + b\sin^2\theta$
- $\left(\sqrt{x^2+a^2} \pm x\right)^n$ or $\left(x \pm \sqrt{x^2-a^2}\right)^n$, put the expression within the bracket $= t$.
- $(x+a)^{-1-\frac{1}{n}}(x+b)^{-1+\frac{1}{n}}$ or $\left(\frac{x+b}{x+a}\right)^{\frac{1}{n}-1}\frac{1}{(x+a)^2}$ $(n \in N, n > 1)$, put $\frac{x+b}{x+a} = t$.
- $\frac{1}{(x+a)^{n_1}(x+b)^{n_2}}$, $n_1, n_2 \in N$ (and > 1), put $(x+a) = t(x+b)$

(ii) Integration by parts

If u and v be two functions of x, then integral product of these two functions is given by

$$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

Usually we use the following preference order for the first function

(Inverse, Logarithmic, Algebraic, Trigonometric, Exponent)

In the above stated order, the function on the left is always chosen as the first function. This rule is called as ILATE.

(iii) Integration by partial fractions

A function of the form $p(x)/\,q(x)$, where $p(x)$ and $q(x)$ are polynomials, is called a rational function. Depending on the nature of $p(x)$ and $q(x)$ integration can be done by the following process:

1. If degree ($p(x)$) < degree ($q(x)$) *i.e.*, $f(x) = \frac{mx+n}{(x-a)(x-b)}$, $a \neq b$ then we write $\frac{mx+n}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, A and B being constants.
2. If degree ($p(x)$) = degree ($q(x)$) or degree ($p(x)$) > degree ($q(x)$) of non-repeated linear factors, *i.e.*, $f(x) = \frac{mx^2+nx+l}{(x-a)(x-b)}$, $a \neq b$ then we write $\frac{mx^2+nx+l}{(x-a)(x-b)} = 1 + \frac{A}{x-a} + \frac{B}{x-b}$
3. If denominator $q(x)$ contains linear factors, some of which are repeated, *i.e.*, integrand is of the form $\frac{p(x)}{(x-a)(x-b)^2}$, then we write the integrand as $\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$.

Algebraic Integrals

- In integrals of the form

$$\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx, \int (px+q)\sqrt{ax^2+bx+c}\,dx$$

Take $px + q = l$ (diff. coefficient of $ax^2 + bx + c$) + m Find l and m by comparing the coefficient of x and constant term on both sides of the identity.

- In integrals of the type

$$\frac{ax^2+bx+c}{px^2+qx+r} \text{ or } \frac{ax^2+bx+c}{\sqrt{px^2+qx+r}}$$

Take $ax^2 + bx + c = M(px^2 + qx + r) + N(2px + q) + R$ Find M, N, and R. The integration reduces to integration of three independent functions.

Integration of Irrational Algebraic Fractions

- Irrational functions of $(ax+b)^{1/n}$ and x can be easily evaluated by the substitution $t^n = ax + b$.

$$\therefore \int f(x,(ax+b)^{1/n})dx = \int f\left(\frac{t^n-b}{a}, t\right)\frac{nt^{n-1}}{a}dt$$

MtG

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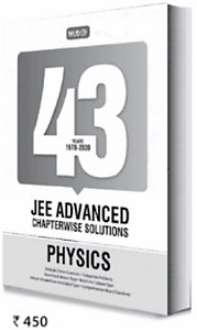

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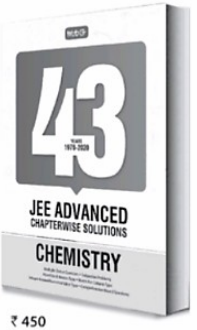

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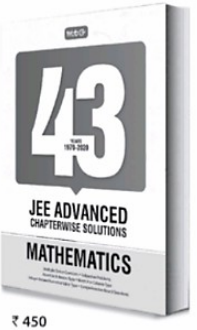

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- In integral of the type $\int \frac{dx}{(x-k)^r\sqrt{ax^2+bx+c}}$, take $x-k=\frac{1}{t}$.

  This substitution will reduce the given integral to $-\int \frac{t^{r-1}dt}{\sqrt{At^2+Bt+C}}$

- To integrate $\frac{1}{(Ax^2+B)\sqrt{Cx^2+D}}$, we first put $x=\frac{1}{t}$,

  $\therefore \int \frac{dx}{(Ax^2+B)\sqrt{Cx^2+D}} = -\int \frac{tdt}{(A+Bt^2)\sqrt{C+Dt^2}}$

  Now, put $C+Dt^2=u^2$, to get $\int \frac{du}{u^2 \pm a^2}$.

- In integral of the type $\int \frac{(ax^2+bx+c)dx}{(dx+e)\sqrt{fx^2+gx+h}}$

  Consider $ax^2+bx+c = A_1(dx+e)(2fx+g) + B_1(dx+e)+C_1$,

  Find A_1, B_1 and C_1 and given integral will reduce to the form

  $$A_1\int \frac{(2fx+g)}{\sqrt{fx^2+gx+h}}dx + B_1\int \frac{dx}{\sqrt{fx^2+gx+h}} + C_1\int \frac{dx}{(dx+e)\sqrt{fx^2+gx+h}}.$$

TRIGONOMETRIC INTEGRALS

- Integrals of the form $\int R(\sin x, \cos x)dx$

  Here R is a rational function of $\sin x$ and $\cos x$. This can be translated into integrals of a rational function by the substitution $\tan\left(\frac{x}{2}\right)=t$.

  In this case

  $$\sin x = \frac{2t}{1+t^2},\quad \cos x = \frac{1-t^2}{1+t^2},$$

  $$x = 2\tan^{-1}t \;;\; dx = \frac{2dt}{1+t^2}$$

  Sometimes, instead of the substitution $\tan\frac{x}{2}=t$, it is more advantageous to make the substitution $\cot\frac{x}{2}=t$.

- **Integrals of the form:**

  (i) $\int \frac{p\cos x + q\sin x + r}{a\cos x + b\sin x + c}dx$

  (ii) $\int \frac{p\cos x + q\sin x}{a\cos x + b\sin x}dx$

  Rule for (i) : In this integral express numerator as l(denominator) + m(derivative of denominator) + n. Find l, m, n and split the integral into sum of three integrals *i.e.*,

  $$l\int dx + m\int \frac{\text{derivative of denominator}}{\text{denominator}}dx + n\int \frac{dx}{a\cos x + b\sin x + c}$$

  Rule for (ii) : Express numerator as l (denominator) + m(derivative of denominator) and find l and m.

- **Integration of the Type:** $\int(\sin^m x\cos^n x)dx$, where m and $n \in$ natural numbers.

(i) If one of them is odd, then substitute for term of even power.
(ii) If both are odd, substitute either of the term.
(iii) If both are even, use trigonometric identities only.

PROBLEMS

Single Correct Answer Type

1. If $I=\int \frac{dx}{e^x+4e^{-x}} = f(x)+c$, then $f(x)$ equals

(a) $2\tan^{-1}(2e^x)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)$

(c) $2\tan^{-1}\frac{e^x}{2}$ (d) $\frac{1}{2}\tan^{-1}(2e^{2x})$

2. $I=\int \frac{\sin x}{\sin x-\cos x}dx$ is equal to

(a) $\frac{x}{2}-\frac{1}{2}\log(\sin x-\cos x)+c$

(b) $\frac{x}{2}+\frac{1}{2}\log(\sin x-\cos x)+c$

(c) $\frac{x}{2}+\frac{1}{2}\log(\sin x+\cos x)+c$

(d) none of these

3. $I=\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$ is equal to

(a) $\frac{\sin 2x}{2}-\sin x+c$ (b) $-\frac{\sin 2x}{2}+\sin x+c$

(c) $-\frac{\sin 2x}{2}-\sin x+c$ (d) $\frac{\sin 2x}{2}+\sin x+c$

4. $I=\int \tan^3 2x\sec 2x dx$ is equal to

(a) $\sec^3 2x + 3\sec 2x + c$ (b) $\frac{1}{6}(\sec^3 2x - 3\sec 2x)+c$

(c) $(\sec^3 2x - 3\sec 2x)+c$ (d) none of these

5. $I=\int \cos^3 x e^{\log(\sin x)}dx$ is equal to

(a) $-\frac{\sin^4 x}{4}+c$ (b) $-\frac{\cos^4 x}{4}+c$

(c) $\frac{e^{\sin x}}{4}+c$ (d) none of these

6. $I=\int \frac{\log(x+1)-\log x}{x(x+1)}dx$ equals

(a) $-\log\left(\frac{x+1}{x}\right)+c$ (b) $-\log\left[\log\left(\frac{x+1}{x}\right)\right]+c$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$ (d) none of these

7. If $\int \frac{1}{x\sqrt{1-x^3}}dx=a\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+b$, then a is equal to

(a) 1/3 (b) 2/3 (c) −1/3 (d) −2/3

8. $I=\int \frac{\cos 2x}{\cos x}dx$ is equal to

(a) $2\sin x+\log(\sec x-\tan x)+c$

(b) $2\sin x-\log(\sec x-\tan x)+c$

(c) $2\sin x+\log(\sec x+\tan x)+c$

(d) $2\sin x-\log(\sec x+\tan x)+c$

9. $\int x^2 \sin x\,dx$ is equal to

(a) $x^2\sin x-2x\cos x+c$

(b) $x^2\sin x+c$

(c) $-x^2\cos x+2x\sin x+2\cos x+c$

(d) $-x^2\sin x-2x\cos x+\sin x+c$

10. $I=\int (1-x)^{23}x\,dx$ is equal to

(a) $\frac{x^{23}}{23}+\frac{x^{24}}{24}+c$ (b) $\frac{(x-1)^{23}}{23}+\frac{(x-1)^{24}}{24}+c$

(c) $\frac{(1-x)^{25}}{25}-\frac{(1-x)^{24}}{24}+c$ (d) none of these

11. $I=\int \frac{dx}{\cos^6 x+\sin^6 x}$ is equal to

(a) $\log_e(\tan x-\cot x)+c$ (b) $\log_e(\cot x+\tan x)+c$

(c) $\tan^{-1}(\tan x-\cot x)+c$ (d) $\tan^{-1}(2\cot 2x)+c$

12. $I=\int \frac{(x+x^{2/3}+x^{1/6})}{x(1+x^{1/3})}dx$ is equal to

(a) $\frac{3}{2}x^{2/3}+6\tan^{-1}(x^{1/6})+c$

(b) $\frac{3}{2}x^{2/3}-6\tan^{-1}(x^{1/6})+c$

(c) $\frac{3}{2}x^{2/3}+\tan^{-1}(x^{1/6})+c$ (d) none of these

13. $I=\int (e^{\log x}+\sin x)\cos x\,dx$ is equal to

(a) $x\sin x+\cos x-\sin^2 x+c$

(b) $x\cos x-\sin^2 x+c$

(c) $x\sin x+\cos x-(\cos^2 x)/2+c$

(d) $x^2\sin x+\cos x-\sin^3 x+c$

14. $\int \frac{1}{4\cos^3 x-3\cos x}dx$ is equal to

(a) $\frac{1}{3}\ln|\sec 3x-\tan 3x|+c$

(b) $\frac{1}{3}\ln|\sec 3x+\tan 3x|+c$

(c) $\frac{1}{4}\ln|\sec 3x+\tan 3x|+c$

(d) $\frac{1}{4}\ln|\sec 3x-\tan 3x|+c$

15. $\int \frac{1}{x[(\ln x)^2+4\ln x-1]}dx$ is equal to

(a) $\frac{1}{\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

(b) $\frac{1}{2}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

(c) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x-2+\sqrt{5}}\right|+c$

(d) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

16. $I=\int \frac{\sin x+\cos x}{9+16\sin 2x}dx$ is equal to

(a) $\frac{1}{40}\ln\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$

(b) $\frac{1}{40}\ln\left|\frac{5-4(\sin x-\cos x)}{5+4(\sin x-\cos x)}\right|+c$

(c) $\frac{1}{40}\ln\left|\frac{5+4(\sin x+\cos x)}{5-4(\sin x+\cos x)}\right|+c$

(d) $\frac{1}{40}\ln\left|\frac{5-4(\sin x+\cos x)}{5+4(\sin x-\cos x)}\right|+c$

17. For what value of a and b, the equation $\int \frac{dx}{1+\sin x}=\tan\left(\frac{x}{2}+a\right)+b$ holds good?

(a) $a=-\frac{5\pi}{4}$, b is any arbitrary constant

(b) $a=\frac{5\pi}{4}$, b is any arbitrary constant

(c) $a=-\frac{\pi}{4}$, b is any arbitrary constant

(d) $a=\frac{\pi}{4}$, b is any arbitrary constant

18. $I=\int\frac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{1+x^2}}dx$ is equal to

(a) $\frac{1}{2}\left[\ln\left(x-\sqrt{x^2+1}\right)\right]^2+c$

(b) $\left[\ln\left(x-\sqrt{x^2+1}\right)\right]^2+c$

(c) $\frac{1}{2}\left[\ln\left(x+\sqrt{x^2+1}\right)\right]^2+c$

(d) $\left[\ln\left(x+\sqrt{x^2+1}\right)\right]^2+c$

19. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ is equal to

(a) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

(b) $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x-\cos x)+c$

(c) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

(d) $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x+\cos x)+c$

20. $\int\frac{\sqrt{5+x^{10}}}{x^{16}}dx$ is equal to

(a) $-\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$ (b) $-\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$

(c) $\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$ (d) $\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$

21. $\int\frac{\left(\sqrt{x^2+1}\right)\{\ln(x^2+1)-2\ln x\}}{x^4}dx$ is equal to

(a) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(b) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(c) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(d) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

22. If $I=\int\frac{dx}{5+4\cos x}=f(x)+c$, then $f(x)=$

(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ (b) $-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$

(c) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ (d) none of these

23. $\int\frac{2^{x+1}-5^{x-1}}{10^x}dx$ is equal to

(a) $\frac{2}{\ln 5}5^x+\frac{1}{5\ln 2}2^x+c$ (b) $\frac{-2}{\ln 5}5^{-x}+\frac{1}{5\ln 2}2^{-x}+c$

(c) $\frac{1}{2\ln 5}5^{-x}-\frac{1}{5\ln 2}2^{-x}+c$

(d) none of these

24. $\int\frac{dx}{x-x^3}=A\ln\left|\frac{x^2}{1-x^2}\right|+c$, then A is equal to

(a) 2 (b) 1/2 (c) 2/3 (d) 1/3

25. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$ is equal to

(a) $\frac{x^2}{2}+\log|x|+c$ (b) $\frac{x^2}{2}+\log|x|+2x+c$

(c) $\frac{1}{3}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3+c$ (d) $\frac{x^2}{2}+\log x-2x+c$

26. $I=\int\frac{(10x^9+10^x\log_e 10)}{(x^{10}+10^x)}dx$ is equal to

(a) $10^x+x^{10}+c$ (b) $10^x-x^{10}+c$
(c) $10^x+x^{10}+c$ (d) $\log_e(10^x+x^{10})+c$

Multiple Correct Answer Type

27. Suppose f is a function that satisfies the equation $f(x+y)=f(x)+f(y)+x^2y+xy^2$ for all real numbers x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then

(a) $f(x)>0$ for $x>0$ and $f(x)<0$ for $x<0$

(b) $f'(0)=1$

(c) $f''(0)=1$

(d) $f'''(x)$ is a constant for all real x

28. If $\int\sqrt{x+\sqrt{x^2+2}}\,dx = A\left\{x+\sqrt{x^2+2}\right\}^{3/2} + \frac{B}{\sqrt{x+\sqrt{x^2+2}}}+c$, then

(a) $A = 1/3$ (b) $B = -2$

(c) $A = 2/3$ (d) $B = -1$

Matrix-Match Type

29. Match the following :

Column-I		Column-II	
A.	If $\int x^2 d(\tan^{-1}x)dx = x - f(x) + c$, then $f(1)$ is equal to	p.	1/8
B.	If $\int x^2 e^{2x}dx = e^{2x}f(x)+c$, then the minimum value of $f(x)$ is	q.	0
C.	If $\int\frac{(x^4+1)}{x(x^2+1)^2}dx = a\log\lvert x\rvert + \frac{b}{x^2+1}+c$ then $(a-b)$ is equal to	r.	$\pi/4$

30. Match the following :

Column-I		Column-II	
A.	$\int\frac{1}{1+\sin x+\cos x}dx =$	p.	$\frac{1}{2}\log\left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right\rvert+c$
B.	$\int\frac{1}{\sqrt{3}\sin x+\cos x}dx =$	q.	$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan(x/2)}{\sqrt{3}}\right)+c$
C.	$\int\frac{1}{1-2\sin x}dx =$	r.	$\log\left\lvert1+\tan\frac{x}{2}\right\rvert+c$
D.	$\int\frac{1}{2+\cos x}dx =$	s.	$\frac{1}{\sqrt{3}}\log\left\lvert\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right\rvert+c$

SOLUTIONS

1. (b) : $I = \int\frac{dx}{e^x+4e^{-x}} = \int\frac{e^x dx}{e^{2x}+4}$

Let $e^x = t \Rightarrow e^x\,dx = dt$

$\Rightarrow I = \int\frac{dt}{t^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c = \frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+c.$

2. (b) : Put $\sin x = A(\sin x - \cos x) + B(\cos x + \sin x)$

$\Rightarrow \sin x = (A+B)\sin x + (B-A)\cos x$

Equating the coefficients of $\sin x$ and $\cos x$, we get

$A + B = 1$ and $B - A = 0 \Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$

$$\therefore I = \int\frac{\frac{1}{2}(\sin x-\cos x)+\frac{1}{2}(\cos x+\sin x)}{\sin x-\cos x}dx$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int\frac{\cos x+\sin x}{\sin x-\cos x}dx$$

$$= \frac{x}{2}+\frac{1}{2}\log(\sin x-\cos x)+c$$

3. (c) : $I = \int\frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\cos\frac{3x}{2}}{\left[1-2\left(2\cos^2\frac{3x}{2}-1\right)\right]\cos\frac{3x}{2}}dx$

$$= \int\frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{3\cos\frac{3x}{2}-4\cos^3\frac{3x}{2}}dx = \int\frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\cos\frac{9x}{2}}dx$$

$$= -\int(\cos 2x+\cos x)dx = -\frac{\sin 2x}{2}-\sin x+c$$

4. (b) : $I = \int\tan^2 2x\tan 2x\sec 2x\,dx$

$= \int(\sec^2 2x-1)\sec 2x\tan 2x\,dx$

Put $\sec 2x = t \Rightarrow 2\sec 2x\tan 2x\,dx = dt$

$$\therefore I = \frac{1}{2}\int(t^2-1)dt = \frac{1}{2}\left(\frac{t^3}{3}-t\right) = \frac{1}{6}(\sec^3 2x-3\sec 2x)+c$$

5. (b)

6. (c) : $I = \int\frac{\log\left(\frac{x+1}{x}\right)}{\left(\frac{x+1}{x}\right)}\cdot\frac{1}{x^2}dx$

Put $t = \frac{x+1}{x} = 1+\frac{1}{x} \Rightarrow dt = -\frac{1}{x^2}dx$

$$\therefore I = -\int(\log t)\frac{1}{t}dt = -\frac{1}{2}(\log t)^2+c$$

$$= -\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$$

7. (a) : Let $I = \int\frac{1}{x\sqrt{1-x^3}}dx$.

Multiply numerator and denominator by x^2.

Put $1-x^3 = t^2 \Rightarrow -3x^2dx = 2t\,dt$

$$\therefore I = \frac{2}{3}\int\frac{dt}{t^2-1} = \frac{2}{3}\cdot\frac{1}{2}\log\left\lvert\frac{t-1}{t+1}\right\rvert+b$$

$$= \frac{1}{3}\log\left\lvert\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right\rvert+b$$

8. (d)

9. (c) : $\int x^2 \sin x\, dx = -x^2\cos x + 2x\sin x + 2\cos x + c$

10. (c)

11. (c) : $I = \int \frac{\sec^6 x}{1+\tan^6 x}dx = \int \frac{(1+\tan^2 x)^2 \sec^2 x dx}{1+\tan^6 x}$

Put $\tan x = p \Rightarrow \sec^2 x\, dx = dp$

$$\therefore I = \int \frac{(1+p^2)^2 dp}{1+p^6} = \int \frac{(1+p^2)}{p^4 - p^2 + 1}dp = \int \frac{p^2\left(1+\frac{1}{p^2}\right)}{p^2\left(p^2+\frac{1}{p^2}-1\right)}dp$$

$$= \int \frac{dk}{k^2+1} = \tan^{-1}(k) + c$$

$$\left[\because \text{ Let } p - \frac{1}{p} = k \Rightarrow \left(1+\frac{1}{p^2}\right)dp = dk\right]$$

$$= \tan^{-1}\left(p - \frac{1}{p}\right) + c = \tan^{-1}(\tan x - \cot x) + c$$

12. (a) : Put $x = p^6 \Rightarrow dx = 6p^5\, dp$

$$\therefore I = \int \frac{6p^5(p^6+p^4+p)}{p^6(1+p^2)}dp = \int \frac{6(p^5+p^3+1)}{(p^2+1)}dp$$

$$= 6\int p^3\, dp + \int \frac{6}{p^2+1}dp$$

$$= \frac{6p^4}{4} + 6\tan^{-1} p = \frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$$

13. (c) : $I = \int x\cos x\, dx + \int \sin x \cos x\, dx$

$$= x\sin x + \cos x - \frac{\cos^2 x}{2} + c$$

14. (b) : Let $I = \int \frac{1}{4\cos^3 x - 3\cos x}dx = \int \frac{1}{\cos 3x}dx$

$$= \int \sec 3x dx = \frac{1}{3}\ln|\sec 3x + \tan 3x| + c$$

15. (d) : Let $I = \int \frac{1}{x[(\ln x)^2 + 4\ln x - 1]}dx$

Put $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore I = \int \frac{dt}{t^2+4t-1} = \int \frac{dt}{(t+2)^2 - (\sqrt{5})^2}$$

$$= \frac{1}{2\sqrt{5}}\ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right| + c = \frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c.$$

16. (a) : Put $t = \sin x - \cos x \Rightarrow t^2 = 1 - \sin 2x$

$$\therefore I = \int \frac{dt}{9+16(1-t^2)} = \int \frac{dt}{25-16t^2}$$

$$= \frac{1}{16}\int \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2} = \frac{1}{16} \times \frac{1}{2\left(\frac{5}{4}\right)}\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right| + c$$

$$= \frac{1}{40}\ln\left|\frac{5+4t}{5-4t}\right| + c = \frac{1}{40}\ln\left|\frac{5+4(\sin x - \cos x)}{5-4(\sin x - \cos x)}\right| + c$$

17. (c) : Here $\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}$

$$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)dx = \frac{1}{2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-1/2} + c$$

$$= -\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) + c = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$$

$\Rightarrow \quad a = -\frac{\pi}{4}$, b is any arbitrary constant.

18. (c) : Put $\ln(x + \sqrt{x^2+1}) = t$

$$\Rightarrow \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)dx = dt$$

$$\therefore I = \int t\, dt = \frac{t^2}{2} + c = \frac{1}{2}\left[\ln\left(x + \sqrt{x^2+1}\right)\right]^2 + c$$

19. (c) : Let $I = \int\left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{2\sin x \cdot \cos x}}dx = \sqrt{2}\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}}dx$$

Put $t = \sin x - \cos x \quad \Rightarrow \quad t^2 = 1 - \sin 2x$

$$\therefore I = \sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}(t) + c$$

$$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + c$$

20. (a) : Let $I = \int \frac{\sqrt{5+x^{10}}}{x^{16}}dx$

Let $1 + \frac{5}{x^{10}} = t \Rightarrow 5\left(\frac{-10}{x^{11}}\right)dx = dt$

$$\therefore I = -\frac{1}{50}\int \sqrt{t}dt = -\frac{1}{50} \times \frac{2}{3}t^{3/2} = -\frac{1}{75}\left(1 + \frac{5}{x^{10}}\right)^{3/2} + c$$

21. (d) : Let $I = \int \frac{\left(\sqrt{\frac{x^2+1}{x^2}}\right)\ln\left(\frac{x^2+1}{x^2}\right)}{x^3}dx$

Put $\frac{x^2+1}{x^2}=t \Rightarrow -\frac{2}{x^3}dx=dt$

$\therefore\ I=-\frac{1}{2}\int\sqrt{t}\ln t\,dt$

$=-\frac{1}{2}\left[(\ln t).\frac{2t^{3/2}}{3}-\frac{2}{3}\int\frac{1}{t}.t^{3/2}dt\right]=\frac{1}{9}t^{3/2}[2-3\ln t]+c$

$=\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

22. (a) : Put $t=\tan\frac{x}{2} \Rightarrow dx=\frac{2dt}{1+t^2}$

$\therefore\ I=\int\frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}=2\int\frac{dt}{t^2+9}=2\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$

$=\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

23. (b) : $\int\frac{2^{x+1}-5^{x-1}}{10^x}dx=\int\left[2\left(\frac{1}{5}\right)^x-\frac{1}{5}\left(\frac{1}{2}\right)^x\right]dx$

$=\frac{2\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}}-\frac{\frac{1}{5}\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}}+c=\frac{-2}{\ln 5}5^{-x}+\frac{1}{5\ln 2}2^{-x}+c$

24. (b) : Let $I=\int\frac{dx}{x-x^3}=\int\frac{dx}{x^3\left(\frac{1}{x^2}-1\right)}$

Put $\frac{1}{x^2}=t \ \Rightarrow\ -\frac{2}{x^3}dx=dt$

$\therefore\ I=-\frac{1}{2}\ln|t-1|+c=-\frac{1}{2}\ln\left|\frac{1-x^2}{x^2}\right|+c$

$=\frac{1}{2}\ln\left|\frac{x^2}{1-x^2}\right|+c \Rightarrow A=\frac{1}{2}$

25. (d) : $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx=\int\left(x+\frac{1}{x}-2\right)dx$

$=\frac{x^2}{2}+\ln|x|-2x+c$

26. (d) : Put $10^x+x^{10}=p \Rightarrow (10^x\log 10+10x^9)dx=dp$

$\therefore\ \ I=\log(x^{10}+10^x)+c$

27. (a, b, d) : Observe that $f(0)=0$ and

$f'(0)=\lim\limits_{h\to 0}\frac{f(h)}{h}=1$

Also $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\frac{f(h)+x^2h+xh^2}{h}=x^2+1$. Hence, $f(x)=\frac{x^3}{3}+x$

28. (a, b) : Put $x+\sqrt{x^2+2}=t \Rightarrow x^2+2=t^2+x^2-2tx$

$\Rightarrow\ x=\frac{1}{2}\left(t-\frac{2}{t}\right)$

$\therefore\ \int\sqrt{x+\sqrt{x^2+2}}\,dx=\frac{1}{2}\int t^{1/2}\left(1+\frac{2}{t^2}\right)dt$

$=\frac{1}{2}\int t^{1/2}dt+\int t^{-3/2}dt=\frac{1}{3}t^{3/2}-\frac{2}{\sqrt{t}}+c$

29. A → r; B → p; C → q

(A) $\int x^2\cdot\frac{1}{1+x^2}dx=x-\tan^{-1}x+c$

$\Rightarrow f(x)=\tan^{-1}x \Rightarrow f(1)=\tan^{-1}1=\frac{\pi}{4}$

(B) $\int x^2e^{2x}dx=\frac{1}{2}x^2\cdot e^{2x}-\int 2x\cdot\frac{1}{2}e^{2x}dx$

$=\frac{1}{2}x^2e^{2x}-\left[x\cdot\frac{1}{2}e^{2x}-\frac{1}{4}e^{2x}\right]=\left(\frac{x^2}{2}-\frac{x}{2}+\frac{1}{4}\right)e^{2x}+c$

(C) $\int\frac{(x^4+1)}{x(x^2+1)^2}dx=\int\frac{((x^2+1)^2-2x^2)}{x(x^2+1)^2}dx$

$=\int\frac{1}{x}dx-2\int\frac{xdx}{(x^2+1)^2}=\log|x|+\frac{1}{x^2+1}+c$

30. A → r; B → p; C → s; D → q

PUZZLE CORNER

ANSWER - DECEMBER 2020

4− 2	24× 6	1	4	15× 3	5
6	12× 2	3	20× 1	5	4
1− 4	3	2	11+ 5	6	3÷ 1
1− 5	4	120× 6	2÷ 2	1	3
3÷ 1	5	4	36× 3	2	6
3	1	11+ 5	6	2− 4	2

CBSE warm-up!

CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Series 8

Three Dimensional Geometry, Linear Programming and Probability

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part-A :

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION-I

All questions are compulsory. In case of internal choices attempt any one.

1. Write the distance of a point $P(a, b, c)$ from x-axis.
2. If a line has direction ratios $2, -1, -2$, then what are its direction cosines?

   OR

   Write the direction cosines of the line joining the points $(1, 0, 0)$ and $(0, 1, 1)$.
3. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
4. Find the vector equation of the plane with intercepts 3, –4 and 2 on x, y and z-axis respectively.
5. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.
6. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$.

OR

If the equation of a line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, then find the numbers to which direction ratios of a line parallel to AB, are proportional.

7. Find the direction cosines of the line that makes equal angles with the three axes in space.

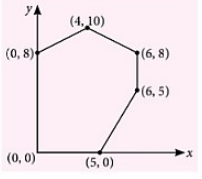
OR

Find the vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

8. Find the value of λ such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2\lambda}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.

9. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). Write the condition, when the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0).

10. The feasible region for an LPP is shown below : Let $z = 3x - 4y$ be the objective function. Write the coordinates where minimum of z occurs.



OR

In solving the LPP :
"Minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" write the redundant constraints.

11. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen. Find the probability of this card to be a card of spade.

12. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

OR

The random variable X has the following probability distribution :

X	0	1	2	3	4	5
$P(X)$	0.1	k	0.1	$3k$	0.3	k

Find the value of k.

13. If A and B are two independent events such that $P(A \cup B) = 0.6$ and $P(A) = 0.2$, then find $P(B)$.

14. If $P(\text{not } E) = 0.36$ and $P(F \mid E) = 0.5$, then find $P(E \cap F)$.

15. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Find the probability that they are of the different colours.

16. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

SECTION-II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. In a school, a football game is to be organised between students of class 11^{th} and 12^{th}. For which, a team from each class is chosen, say T_1 be the team of class 11^{th} and T_2 be the team of class 12^{th}. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively.

Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game.
Let X and Y denote the total points scored by team T_1 and T_2, respectively, after two games.
Based on the above information, answer the following questions:

(i) $P(T_2$ winning a match against $T_1)$ is equal to
(a) 1/2 (b) 1/6
(c) 1/3 (d) none of these

(ii) $P(T_2$ drawing a match against $T_1)$ is equal to

(a) 1/2 (b) 1/3

(c) 1/6 (d) 2/3

(iii) $P(X > Y)$ is equal to

(a) 1/4 (b) 5/12

(c) 1/2 (d) 7/12

(iv) $P(X = Y)$ is equal to

(a) 11/36 (b) 1/3

(c) 13/36 (d) 1/2

(v) $P(X + Y = 12)$ is equal to

(a) 0 (b) 5/12

(c) 13/36 (d) 7/12

18. Suppose you visit to a hotel with your family and you observe that floor of a hotel is made up of mirror polished Kota stone. Also, there is a large crystal chandelier attached at the ceiling of the hotel. Consider the floor of the hotel as a plane having equation $3x - y + 4z = 2$ and crystal chandelier as the point $(3, -2, 1)$.

Based on the above information, answer the following questions :

(i) The d.r.'s of the perpendicular from the point $(3, -2, 1)$ to the plane $3x - y + 4z = 2$, is

(a) <3, 1, 4 > (b) <3, – 1, 4 >

(c) <4, 1, 3> (d) <4, –1, 3>

(ii) The length of the perpendicular from the point $(3, -2, 1)$ to the plane $3x - y + 4z = 2$, is

(a) $\sqrt{13}$ units (b) $\frac{1}{2}\sqrt{13}$ units

(c) $\sqrt{\frac{13}{2}}$ units (d) $\frac{13}{\sqrt{2}}$ units

(iii) The equation of the perpendicular from the point $(3, -2, 1)$ to the plane $3x - y + 4z = 2$, is

(a) $\frac{x-3}{3} = \frac{y-2}{-1} = \frac{z-1}{4}$

(b) $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4}$

(c) $\frac{x+3}{3} = \frac{y+2}{-1} = \frac{z-1}{4}$

(d) None of these

(iv) The foot of the perpendicular drawn from the point $(3, -2, 1)$ to the plane $3x - y + 4z = 2$, is

(a) $\left(\frac{3}{2}, \frac{-3}{2}, -1\right)$ (b) $\left(\frac{-3}{2}, \frac{3}{2}, -1\right)$

(c) $\left(\frac{3}{2}, \frac{3}{2}, -1\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$

(v) The image of the point $(3, -2, 1)$ in the given plane is

(a) (0, 1, 3) (b) (0, –1, 3)

(c) (0, 1, –3) (d) (0, –1, –3)

PART-B

SECTION-III

All questions are compulsory. In case of internal choices attempt any one.

19. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

20. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

21. Prove that if E and F are independent events, then the events E' and F' are also independent.

OR

The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

22. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

23. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

OR

Find the coordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX-plane.

24. Find the distance between the lines given by $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$.

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25. Find the cartesian equation of the plane passing through a point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$.

OR

Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.

26. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

27. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.

28. Suppose that two cards are drawn at random from a deck of 52 cards. Let X be the number of aces obtained. Then, find the probability distribution of X.

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

30. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

31. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. If an orator is chosen at random, find the probability that a male person is selected. Assume that there are equal number of men and women.

OR

A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

32. Solve the following LPP graphically :
Minimise $Z = 5x + 10y$
Subject to constraints $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$ and $x, y \ge 0$.

33. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

Find the unit vector perpendicular to the plane ABC where the position vectors of A, B and C are $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}+3\hat{k}$ respectively.

34. Show that the lines
$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$;
$\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$ are intersecting.
Hence find their point of intersection.

35. Maximise $Z = x + 2y$
subject to the constraints :
$x + 2y \ge 100$
$2x - y < 0$
$2x + y \le 200$
$x, y \ge 0$
Solve the above LPP graphically.

SECTION-V

All questions are compulsory. In case of internal choices attempt any one.

36. Find the vector and cartesian equation of the line through the point $\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the lines
$\vec{r}=2\hat{i}-3\hat{j}+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and
$\vec{r}=3\hat{i}-5\hat{j}+\mu(\hat{i}+\hat{j}+\hat{k})$.

OR

Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line, $\vec{r}=-\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$.
Also find the image of P in this line.

37. Find the minimum value of $Z = 3x + 4y + 270$ subject to the constraints

$x + y \le 60 ; x + y \ge 30 ; x \le 40, y \le 40 ; x \ge 0, y \ge 0$

OR

Maximise $z = 8x + 9y$ subject to the constraints given below :

$2x + 3y \le 6, 3x - 2y \le 6, y \le 1; x, y \ge 0$

38. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian?

OR

Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

SOLUTIONS

1. We have, equation of x-axis as $y = 0, z = 0$

$\therefore$ Distance of $P(a, b, c)$ from x-axis

$= \sqrt{(a-a)^2 + b^2 + c^2} = \sqrt{b^2 + c^2}$ units.

2. The direction ratios of the given line are 2, –1, –2.

Let $a = 2, b = -1, c = -2$

$\Rightarrow \sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3$

$\therefore$ The direction cosines of given line are

$\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$.

OR

The direction cosines of the line joining the points (1, 0, 0) and (0, 1, 1) are $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

3. The eq. of given line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ ...(i)

$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} \Rightarrow \frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$

As $\sqrt{2^2 + (-6)^2 + 3^2} = 7$

$\therefore$ D.c's. of (i) are $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$.

4. Intercepts are $a = 3, b = -4, c = 2$

$\therefore$ The intercept form of the plane is

$\frac{x}{3} + \left(\frac{y}{-4}\right) + \frac{z}{2} = 1 \Rightarrow 4x - 3y + 6z = 12$

So, the vector equation of the plane is

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$

$\Rightarrow \vec{r} \cdot (4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$

5. Let α, β and γ be the angles made by $\vec{n}$ with x, y and z-axis, respectively.

Given $\alpha = \beta = \gamma \Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow$ $l = m = n$, where l, m, n are direction cosines of $\vec{n}$.

But $l^2 + m^2 + n^2 = 1 \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

So, $l = m = n = \pm\frac{1}{\sqrt{3}}$

$\therefore$ The normal form of the plane is $lx + my + nz = d$

$\Rightarrow \pm\frac{1}{\sqrt{3}}x \pm \frac{1}{\sqrt{3}}y \pm \frac{1}{\sqrt{3}}z = 5\sqrt{3} \Rightarrow \pm x \pm y \pm z = 15$

6. Length of the $\perp$ drawn from $O(0, 0, 0)$ to the plane $2x - 3y + 6z + 21 = 0$ is

$\left|\frac{2\times 0 - 3\times 0 + 6\times 0 + 21}{\sqrt{2^2 + (-3)^2 + 6^2}}\right| = \frac{21}{7} = 3$ units.

OR

Since direction ratios of line AB are 1, –2, 4, therefore the direction ratios of line parallel to AB will be proportional to 1, –2 and 4.

7. Since, $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \pm\frac{1}{\sqrt{3}}$.

OR

The vector form of $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ is

$\vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$\therefore$ Required equation in vector form is

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

8. Direction ratios of the given lines are $(1, 3, 2\lambda)$ and $(-3, 5, 2)$ respectively. Since, the lines are at right angles, so $(1) \times (-3) + (3) \times (5) + 2(2\lambda) = 0$

$\Rightarrow -3 + 15 + 4\lambda = 0 \Rightarrow \lambda = -3$

9. Since, maximum value of $z = ax + by$ occurs at both (2, 4) and (4, 0).

$\therefore \quad 2a + 4b = 4a + 0 \Rightarrow 4b = 2a \Rightarrow 2b = a$

10. We know that minimum of objective function occurs at corner points.

Corner points	Value of $z = 3x - 4y$
(0, 0)	0
(5, 0)	15
(6, 5)	−2
(6, 8)	−14
(4, 10)	−28
(0, 8)	−32 ← Minimum

OR

When $x \geq 6$ and $y \geq 2$, then

$2x + y \geq 2 \times 6 + 2$, *i.e.*, $2x + y \geq 14$

Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region

$\{(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$

11. Let A be the event that the card is a spade and B be the event that the picked card is a queen.

We have a total of 13 spades and 4 queen cards.

Also only one queen is from spade.

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$$

12. Let A, B, C be respectively the events of solving problem by three students and $P(A)$, $P(B)$, $P(C)$ be their probability of solving the problem respectively.

$$\therefore \quad P(A) = \frac{1}{3},\ P(B) = \frac{1}{4} \text{ and } P(C) = \frac{1}{6}$$

Required probability $= 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$

$= 1 - P(\overline{A})\ P(\overline{B})\ P(\overline{C})$ ($\because A, B, C$ are independent

$\therefore\ \overline{A}, \overline{B}, \overline{C}$ are also independent)

$= 1 - [1 - P(A)]\ [1 - P(B)]\ [1 - P(C)]$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} = 1 - \frac{5}{12} = \frac{7}{12}$$

OR

$$\because \sum_{x=0}^{5} P(X = x) = 1$$

$\Rightarrow \quad P(X = 0) + P(X = 1) + ... + P(X = 5) = 1$

$\Rightarrow \quad 0.1 + k + 0.1 + 3k + 0.3 + k = 1 \Rightarrow 0.5 + 5k = 1$

$\Rightarrow \quad 5k = 1 - 0.5 = 0.5 \Rightarrow k = 0.1$

13. If A and B are two independent events, then

$P(A \cap B) = P(A) \times P(B)$

It is given that $P(A \cup B) = 0.6$, $P(A) = 0.2$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$\Rightarrow \quad 0.6 = 0.2 + P(B)(1 - 0.2) \Rightarrow 0.4 = P(B)\ (0.8)$

$$\Rightarrow \quad P(B) = \frac{0.4}{0.8} \Rightarrow P(B) = \frac{1}{2} = 0.5$$

14. Clearly, $P(E) = 1 - P(\overline{E}) = 1 - 0.36 = 0.64$

Now, $P(F \mid E) = 0.5 \Rightarrow \dfrac{P(E \cap F)}{P(E)} = 0.5$

$\Rightarrow \quad P(E \cap F) = P(E) \times 0.5 = 0.64 \times 0.5 = 0.32$

15. Total number of possible outcomes $= {}^6C_2 = 15$

Number of favourable outcomes $= {}^2C_1\ {}^4C_1 = 2 \times 4 = 8$

$$\therefore \quad \text{Required probability} = \frac{8}{15}$$

16. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$.

$$\text{Now, } P(E) = \frac{10}{15},\ P(F \mid E) = \frac{9}{14}$$

By multiplication rule of probability, we have

$$P(E \cap F) = P(E) \cdot P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$$

17. **(i) (c) :** Clearly, $P(T_2$ winning a match against $T_1)$ $= P(T_1$ losing$) = 1/3$

(ii) (c) : Clearly, $P(T_2$ drawing a match against $T_1)$ $= P(T_1$ drawing$) = 1/6$

(iii) (b) : According to given information, we have the following possibilities for the value of X and Y.

X	6	4	3	2	1	0
Y	0	1	3	2	4	6

Now, $P(X > Y) = P(X = 6, Y = 0) + P(X = 4, Y = 1)$

$= P(T_1$ win$)\ P(T_1$ win$) + P(T_1$ win$)\ P($match draw$)$

$+ P($match draw$)\ P(T_1$ win$)$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{2} = \frac{3+1+1}{12} = \frac{5}{12}$$

(iv) (c) : $P(X = Y) = P(X = 3, Y = 3) + P(X = 2, Y = 2)$

$= P(T_1$ win$)\ P(T_2$ win$) + P(T_2$ win$)\ P(T_1$ win$)$

$+ P($match draw$)\ P($match draw$)$

$$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{36} = \frac{13}{36}$$

(v) (a) : From the given information, it is clear that maximum sum of X and Y can be 6, therefore $P(X + Y = 12) = 0$

18. (i) (b) : Equation of plane is $3x - y + 4z = 2$

$\therefore$ D.R.'s of normal to the plane are <3, –1, 4>, which is also the D.R.'s of perpendicular from the point (3, –2, 1) to the given plane.

(ii) (c) : Required length = Perpendicular distance from (3, –2, 1) to the plane $3x - y + 4z = 2$

$$= \left|\frac{3\times3-(-2)+4(1)-2}{\sqrt{3^2+(-1)^2+(4)^2}}\right| = \left|\frac{9+2+4-2}{\sqrt{9+1+16}}\right| = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}} \text{ units}$$

(iii) (b) : The equation of perpendicular from the point (x_1, y_1, z_1) to the plane $ax + by + cz = d$ is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Here, $x_1 = 3, y_1 = -2, z_1 = 1$ and $a = 3, b = -1, c = 4$

$\therefore$ Required equation is $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4}$.

(iv) (a) : Let the coordinate of image of (3, –2, 1) be $Q(3r + 3, -r - 2, 4r + 1)$

Let R be the mid-point of PQ, then coordinate of R be

$$\left(\frac{3r+6}{2}, \frac{-r-4}{2}, 2r+1\right)$$

Since, R lies on the plane $3x - y + 4z = 2$

$$\therefore \quad 3\left(\frac{3r+6}{2}\right) - \left(\frac{-r-4}{2}\right) + 4(2r+1) = 2$$

$$\Rightarrow \quad 26r = -26 \Rightarrow r = -1$$

Thus, the coordinates of R be $\left(\frac{3}{2}, \frac{-3}{2}, -1\right)$

(v) (d) : Putting the value of $r = -1$ in $Q(3r + 3, -r - 2, 4r + 1)$, we get $Q(0, -1, -3)$, which is the required image of given point.

19. Total number of students = 8

The number of ways to select 4 students out of 8 students

$$= {}^8C_4 = \frac{8!}{4!\,4!} = 70$$

The number of ways to select 2 boys and 2 girls

$$= {}^3C_2 \times {}^5C_2 = \frac{3!}{2!1!} \times \frac{5!}{2!3!} = 3 \times 10 = 30$$

$\therefore$ Required probability $= \frac{30}{70} = \frac{3}{7}$

20. E : 'a total of 8' and F : 'red die resulted in a number less than 4'

i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$

i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

Hence, $E \cap F = \{(5, 3), (6, 2)\}$, $P(E) = 5/36$, $P(F) = 18/36$, $P(E \cap F) = 2/36$

$\therefore$ Required probability $= P(E/F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}$$

21. Since, E and F are independent events.

$\therefore \quad P(E \cap F) = P(E)\,P(F)$ ...(i)

Now, $P(E' \cap F') = 1 - P(E \cup F)$

$[\because P(E' \cap F') = P((E \cup F)')]$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - P(E) - P(F) + P(E)\,P(F)$ [Using (i)]

$= (1 - P(E))\,(1 - P(F)) = P(E')\,P(F')$

Hence, E' and F' are independent events.

OR

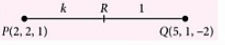
Let G be the event of a green signal.

Required probability $= P(GGG') + P(GG'G)$

$$= \left(\frac{3}{10}\right)^2 \frac{7}{10} + \frac{7}{10} \cdot \left(\frac{3}{10}\right)^2 = \frac{9}{100} \cdot \frac{7}{10} + \frac{7}{10} \cdot \frac{9}{100}$$

$$= \frac{63}{1000} + \frac{63}{1000} = \frac{126}{1000}$$

22. Given that $P(2, 2, 1)$ and $Q(5, 1, -2)$



CBSE Board Exam 2021 to be held in written mode only, not online

The Central Board of Secondary Education (CBSE) on 2nd December said that no final decision has been taken regarding the dates for conduct of board exams 2021. The board also said that consultations with stakeholders are still in process before taking a final call.

CBSE spokesperson Rama Sharma informed that the final dates for conduct of board exams has not been taken, but they will be conducted in written mode and not online mode whenever they are held. Sharma further said that the exams will be conducted following all COVID-related protocols. Informing about practical exams, Sharma said in case students are not able to do practicals in classes before the exams, alternatives to practical exams will have to be explored.

According to reports education ministry is planning to consult students and their parents regarding holding of examinations as a fresh cycle of key tests is set to begin with the onset of the New Year. The official said that in opinion of education ministry the examination calendar for 2021 will need to be tweaked as Covid-19 situation is still continuing.

Let the point R on the line PQ, divides the line in the ratio $k:1$. And x-coordinate of point R on the line is 4. So, by section formula

$$4=\frac{5k+2}{k+1} \Rightarrow k=2$$

Now, z-coordinate of point R

$$=\frac{-2k+1}{k+1}=\frac{-2\times2+1}{2+1}=-1$$

23. The given lines are

$$l_1:\frac{x-0}{1}=\frac{y-0}{-1}=\frac{z-0}{\frac{1}{k}};\ l_2:\frac{x-2}{1}=\frac{y+\frac{1}{2}}{\frac{1}{2}}=\frac{z-1}{-1}$$

$\because$ l_1 is perpendicular to l_2

$$\therefore \quad 1(1)+(-1)\left(\frac{1}{2}\right)+\left(\frac{1}{k}\right)(-1)=0$$

$$\Rightarrow \quad 1-\frac{1}{2}-\frac{1}{k}=0 \Rightarrow \frac{1}{2}=\frac{1}{k} \Rightarrow k=2$$

OR

The equation of line through $A(-1,1,-8)$ and $B(5,-2,10)$ is

$$\frac{x+1}{5+1}=\frac{y-1}{-2-1}=\frac{z+8}{10+8}$$

i.e., $\frac{x+1}{6}=\frac{y-1}{-3}=\frac{z+8}{18}=k$(say) ...(i)

Any point on (i) is given by $(6k-1,-3k+1,18k-8)$.
We know that the coordinates of any point on the ZX-plane is given by $(x,0,z)$.
$\therefore$ $-3k+1=0 \Rightarrow k=1/3$
Thus the coordinates of the point where the line joining A and B crosses the ZX-plane are

$$\left(6\times\frac{1}{3}-1,0,18\times\frac{1}{3}-8\right)=(1,0,-2)$$

24. The given lines are parallel.

Here, $\vec{a}_1=\hat{i}+\hat{j},\vec{a}_2=2\hat{i}-3\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=(2\hat{i}-3\hat{k})-(\hat{i}+\hat{j})=\hat{i}-\hat{j}-3\hat{k}$

$$\vec{b}\times(\vec{a}_2-\vec{a}_1)=\hat{i}(6+3)-\hat{j}(-3-3)+\hat{k}(-1+2)$$
$$=9\hat{i}+6\hat{j}+\hat{k}$$

$$|\vec{b}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$$

Shortest distance $=\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}=\frac{|9\hat{i}+6\hat{j}+\hat{k}|}{\sqrt{14}}$

$$=\frac{1}{\sqrt{14}}\sqrt{(9)^2+(6)^2+(1)^2}=\sqrt{\frac{118}{14}}=\sqrt{\frac{59}{7}}\ \text{units}$$

25. Here, $(x_1,y_1,z_1)=(2,3,4)$, $a=2$, $b=1$, $c=-2$
Cartesian equation is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$
$$\Rightarrow \quad 2(x-2)+1(y-3)-2(z-4)=0$$
$$\Rightarrow \quad 2x-4+y-3-2z+8=0 \Rightarrow 2x+y-2z=-1$$

OR

Any point on line

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$$

is $(2+3\lambda,-1+4\lambda,2+12\lambda)$ ...(i)

If it lies on the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$,
then, $1(2+3\lambda)-1(-1+4\lambda)+1(2+12\lambda)=5$
$$\Rightarrow \quad 2+3\lambda+1-4\lambda+2+12\lambda=5$$
$$\Rightarrow \quad 11\lambda=0 \Rightarrow \lambda=0$$
Putting $\lambda=0$ in (i), we get $(2,-1,2)$
Required distance $=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$
$=\sqrt{9+16+144}=13$ units

26. If all the 36 elementary events of the experiment are considered to be equally likely, we have

$$P(A)=\frac{18}{36}=\frac{1}{2} \text{ and } P(B)=\frac{18}{36}=\frac{1}{2}$$

Also, $P(A\cap B)=P$(odd number on both throws)

$$=\frac{9}{36}=\frac{1}{4}$$

Now, $P(A)P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Clearly, $P(A\cap B)=P(A)\times P(B)$
Thus, A and B are independent events.

27. Let E be the event that A is coming in time;
$P(E)=3/7$
and F be the event that B is coming in time, $P(F)=5/7$
Also E and F are given to be independent events.
$\therefore$ Probability of only one of them coming to the school in time $=P(E)\cdot P(\bar{F})+P(\bar{E})\cdot P(F)$

$$=\frac{3}{7}\cdot\left(1-\frac{5}{7}\right)+\left(1-\frac{3}{7}\right)\cdot\frac{5}{7}=\frac{3}{7}\cdot\frac{2}{7}+\frac{4}{7}\cdot\frac{5}{7}=\frac{26}{49}$$

28. Total number of aces $=4$
Also, X can take the values 0, 1, 2

$$\therefore \quad P(X=0)=\frac{{}^4C_0\times{}^{48}C_2}{{}^{52}C_2}=\frac{188}{221},$$

$$P(X=1)=\frac{{}^4C_1\times{}^{48}C_1}{{}^{52}C_2}=\frac{32}{221}$$

and $P(X=2)=\frac{{}^4C_2\times{}^{48}C_0}{{}^{52}C_2}=\frac{1}{221}$

The probability distribution of X is as follows :

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

29. Probability of choosing bag $A = P(A) = \frac{2}{6} = \frac{1}{3}$

Probability of choosing bag $B = P(B) = \frac{4}{6} = \frac{2}{3}$

Let E_1 and E_2 be the events of drawing a red and a black ball from bag A and B respectively.

$\therefore \quad P(E_1) = \frac{6\times4}{{}^{10}C_2}$ and $P(E_2) = \frac{7\times3}{{}^{10}C_2}$

$\therefore$ Required probability

$= P(A) \times P(E_1) + P(B) \times P(E_2)$

$= \frac{1}{3}\times\frac{6\times4}{{}^{10}C_2} + \frac{2}{3}\times\frac{7\times3}{{}^{10}C_2} = \frac{1}{3}\times\frac{8}{15} + \frac{2}{3}\times\frac{7}{15} = \frac{8}{45} + \frac{14}{45} = \frac{22}{45}$

30. *Refer to Answer 21, page no. 314 of MTG CBSE Champion Mathematics, Class 12.*

31. Let E_1, E_2 and A denote the events defined as follows :

E_1 = person selected is man

E_2 = person selected is woman

A = person selected is good orator

We have, $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

Now, $P(A|E_1) = \frac{5}{100}$ and $P(A|E_2) = \frac{25}{1000}$

Required probability is

$$P(E_1|A) = \frac{P(E_1)\times P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

[Using Bayes' theorem]

$$= \frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100} + \frac{1}{2}\times\frac{25}{1000}} = \frac{50}{75} = \frac{2}{3}$$

OR

Let E_1 be the event of choosing a biased coin and E_2 be the event of choosing an unbiased coin.

$\Rightarrow \quad P(E_1) = P(E_2) = \frac{1}{2}$

Given, probability of biased coin has the chance of showing heads is 60%.

$\therefore$ Probability of biased coin has the chance of showing tail is 40%.

Let A be the event of showing tails.

$\therefore \quad P(A|E_1) = \frac{40}{100} = \frac{2}{5}$; $P(A|E_2) = \frac{1}{2}$

Using Bayes' theorem, we get

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)(A|E_1) + P(E_2)(A|E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{2}{5} + \frac{1}{2}\times\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{5} + \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{20}} = \frac{5}{9}$$

32. *Refer to Answer 5, page no. 290-291 of MTG CBSE Champion Mathematics, Class 12.*

33. Any point on the line

$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r$ (say) ...(i)

is $(3r - 1, 5r - 3, 7r - 5)$.

Any point on the line

$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = k$ (say) ...(ii)

is $(k + 2, 3k + 4, 5k + 6)$

For lines (i) and (ii) to intersect, we must have

$3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$

$\Rightarrow \quad 3r - k = 3 ; 5r - 3k = 7 ; 7r - 5k = 11$

On solving these, we get $r = \frac{1}{2}$, $k = -\frac{3}{2}$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$.

OR

Here the position vectors of A, B and C are respectively $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$.

$\therefore \quad \overrightarrow{AB} = (\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{AC} = (2\hat{i} + 3\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = 0\cdot\hat{i} + \hat{j} + 2\hat{k}$

A vector normal to the plane containing points A, B and C is

$$\vec{n} = \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ The required unit vector

$$= \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{3^2 + 2^2 + (-1)^2}} = \frac{1}{\sqrt{14}}(3\hat{i} + 2\hat{j} - \hat{k})$$

34. The given lines are

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$\Rightarrow \quad \vec{r} = (3 + \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (2\lambda - 4)\hat{k}$ ...(i)

and $\vec{r} = (5+3\mu)\hat{i} + (2\mu - 2)\hat{j} + 6\mu\hat{k}$ ...(ii)

If these lines intersect, they must have a common point. So, we must have

$(3+\lambda)\hat{i} + (2+2\lambda)\hat{j} + (2\lambda - 4)\hat{k}$

$= (5+3\mu)\hat{i} + (2\mu - 2)\hat{j} + 6\mu\hat{k}$

$\Rightarrow \quad 3 + \lambda = 5 + 3\mu \Rightarrow \lambda - 3\mu = 2,$

$2 + 2\lambda = 2\mu - 2 \Rightarrow \lambda - \mu = -2,$

and $2\lambda - 4 = 6\mu \Rightarrow \lambda - 3\mu = 2 \Rightarrow \lambda = -4, \mu = -2.$

$\therefore$ The given lines intersect and their point of intersection is $(-1, -6, -12)$.

35. *Refer to Answer 6, page no. 291 of MTG CBSE Champion Mathematics, Class 12.*

36. Here we need to find, the equation of the line through the point (1, 1, −3) and perpendicular to the lines

$$\frac{x-2}{2} = \frac{y+3}{1} = \frac{z-0}{-3} \quad \ldots(i)$$

and $$\frac{x-3}{1} = \frac{y+5}{1} = \frac{z-0}{1} \quad \ldots(ii)$$

Let the direction ratios of required line are a, b, c. Then, equations of this line is given by

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z+3}{c} \quad \ldots(iii)$$

Direction ratios of line (i) are 2, 1, −3 and line is perpendicular to line (iii) having direction ratios a, b, c

$\therefore \quad 2a + b - 3c = 0 \quad \ldots(iv)$

Similarly $a + b + c = 0 \quad \ldots(v)$

Solving equation (iv) and (v), we get

$$\frac{a}{4} = \frac{-b}{5} = \frac{c}{1}$$

$\therefore$ From (iii), required equation of the line is

$$\frac{x-1}{4} = \frac{y-1}{-5} = \frac{z+3}{1}.$$

Its vector equation is $\vec{r} = (\hat{i} + \hat{j} - 3\hat{k}) + \lambda(4\hat{i} - 5\hat{j} + \hat{k})$

OR

Refer to Answer 25, page no. 264 of MTG CBSE Champion Mathematics, Class 12.

37. Converting inequations into equations, we get

$x + y = 60 \quad \ldots(i)$

$x = 40 \quad \ldots(ii)$

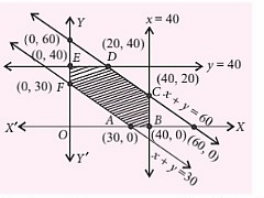
$y = 40 \quad \ldots(iii)$

$x + y = 30 \quad \ldots(iv)$

$x = 0 \quad \ldots(v)$

and $y = 0 \quad \ldots(vi)$

Let us draw the graph of equations (i) to (vi). The feasible region is shown in figure.



The coordinates of the corner points of the feasible region are $A(30, 0)$, $B(40, 0)$, $C(40, 20)$, $D(20, 40)$, $E(0, 40)$ and $F(0, 30)$.

Corner points	Value of $Z = 3x + 4y + 270$
$A(30, 0)$	360 ← Minimum
$B(40, 0)$	390
$C(40, 20)$	470
$D(20, 40)$	490
$E(0, 40)$	430
$F(0, 30)$	390

From the table, the minimum value of Z is 360, which is attained at the point $A(30, 0)$.

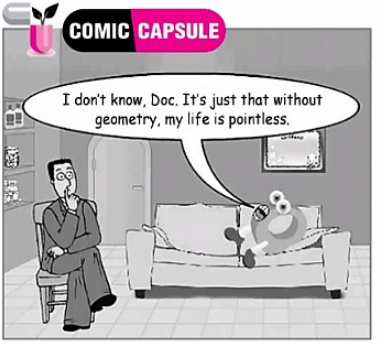
OR

Refer to Answer 8, page no. 291-292 of MTG CBSE Champion Mathematics, Class 12.

38. *Refer to Answer 50, page no. 320 of MTG CBSE Champion Mathematics, Class 12.*

OR

Refer to Answer 49, page no. 320 of MTG CBSE Champion Mathematics, Class 12.



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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 | **Series 8 : Differential Equations** | **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $\frac{dx}{dy} = (e^y - x)$, where $y(0) = 0$, then y is expressed explicitly as

(a) $\frac{1}{2}\ln(1+x^2)$ (b) $\ln(1+x^2)$

(c) $\ln\left(x+\sqrt{1+x^2}\right)$ (d) $\ln\left(x+\sqrt{1-x^2}\right)$

2. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by (a is the arbitrary constant)

(a) $x^n + n^2y$ = constant (b) $ny^2 + x^2$ = constant

(c) $n^2x + y^n$ = constant (d) $n^2x - y^n$ = constant

3. Let $f(x)$ be continuously differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$ for each $x > 0$, then $f(x)$ is

(a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $-\frac{1}{3x} + \frac{4x^2}{3}$

(c) $-\frac{1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$

4. If $xdy - ydx + x\cos(\log_e x)dx = 0$, $y(1) = 1$, then $y(e) =$

(a) $e(1 - \cos 1)$ (b) $e(1 - \sin 1)$

(c) $e(1 + \cos 1)$ (d) $e(1 + \sin 1)$

5. Let $y' = \frac{4y^2 + 4xy + x^2}{4x^2}$ and $y(1) = 0$, then $y(e^{\pi/2}) =$

(a) $\frac{1}{2}e^{\frac{\pi}{2}}$ (b) $e^{\frac{\pi}{2}}$ (c) $\frac{1}{4}e^{\frac{\pi}{2}}$ (d) $\frac{\pi}{2}$

6. Solution of the differential equation

$\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\}dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\}dy = 0$ is

(a) $\ln\left|\frac{x}{y}\right| + \frac{xy}{(x-y)} = c$ (b) $\ln|xy| + \frac{xy}{(x-y)} = c$

(c) $\frac{xy}{(x-y)} = ce^{x/y}$ (d) $\frac{xy}{(x-y)} = ce^{xy}$

One or More Than One Option(s) Correct Type

7. Let f be a non negative function defined on the interval $[0, \pi/2]$. If

$\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t\, dt$

and $f(0) = 1$, then

(a) $f(0) = 1$ is the maximum value of f

(b) $f(0) = 1$ is the minimum value of f

(c) $f(\pi/4) = \frac{3\sqrt{2}}{2} - 1$

(d) $f(\pi/3) = 1$

8. If $f(x)$, $g(x)$ is twice differential functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then

(a) $f(4) - g(4) = 10$

(b) $|f(x) - g(x)| < 2 \Rightarrow -2 < x < 0$

(c) $f(x) - g(x) = 2x$ has real root

(d) None of these

9. For the differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ (a is a constant), its

(a) order is 2 (b) order is 3

(c) degree is 2 (d) degree is 3

10. The solution of $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ are

(a) $y + e^{-x}$ = constant (b) $x = \log(c + y)$

(c) $y = e^{-x}$ + constant (d) $y = e^x$ + constant

11. Solution of the differential equation $\frac{dy}{dx} = \frac{x+y+7}{2x+2y+3}$ is equal to

(a) $6(x+y) + 11\ln(3x+3y+10) = 9x + c$

(b) $6(x+y) - 11\ln(3x+3y+10) = 9x + c$

(c) $6(x+y) - 11\ln\left(x+y+\frac{10}{3}\right) = 9x + c$

(d) none of these

12. A particle falls in a medium whose resistance is proportional to the square of the velocity of the particle. If the differential equation of the free fall is $\frac{dv}{dt} = g - kv^2$ (k is constant), then

(a) $v = 2\sqrt{\frac{g}{k}}\frac{e^{2t\sqrt{gk}}+1}{e^{2t\sqrt{gk}}-1}$

(b) $v = \sqrt{\frac{g}{k}}\frac{e^{2t\sqrt{gk}}-1}{e^{2t\sqrt{gk}}+1}$

(c) $v \to 0$ as $t \to \infty$

(d) $v \to \sqrt{\frac{g}{k}}$ as $t \to \infty$

13. $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ has the solution

(a) $y - \frac{c}{1+\cos x} = 0$ (b) $y = \frac{c}{1-\cos x}$

(c) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$ (d) $x = 2\cos^{-1}\left(\frac{c}{2y}\right)$

Comprehension Type

Consider the curve passing through the point (1, 0) and for which the length of the subnormal at any point of the curve is equal to the arithmetic mean of the co-ordinates of this point. Then

14. The equation of the curve is

(a) $(y-x)^2(x+2y) = 1$ (b) $(y+x)^2(x+2y) = 1$

(c) $(y-x)^2(2y-x) = 1$ (d) $(y+x)^3(x+2y) = 1$

15. The curve meets y = axis at

(a) $(0, (1/2)^{1/4})$ (b) $(0, (1/2)^{1/3})$

(c) $(0, (1/2)^{1/2})$ (d) none of these

Matrix Match Type

16. Match the following columns :

	Column-I		Column-II
P.	Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is	1.	7
Q.	If real value of m for which the substitution, $y = u^m$ will transform the differential equation, $2x^4 y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation, then the value of $2m$ is	2.	2
R.	If the solution of differential equation $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$, then $\lvert m+n\rvert$ is	3.	3

	P	Q	R
(a)	3	1	2
(b)	2	3	1
(c)	1	2	3
(d)	2	1	3

Numerical Value Type

17. If $x\cos(y/x)(y\,dx + x\,dy) = y\sin(y/x)(x\,dy - y\,dx)$, $y(1) = 2\pi$, then the value of $4\frac{y(4)}{\pi}\cos\left(\frac{y(4)}{4}\right)$ is _____.

18. Let $y = y(t)$ be a solution to the differential equation $y' + 2ty = t^2$, then $16\lim_{t\to\infty}\frac{y}{t}$ is _____.

19. If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3} - 13\frac{dy}{dx}}{y} = k$, then the value of $k/2$ is _____.

20. If the curve satisfying $(xy^4 + y)\,dx - x\,dy = 0$ passes through (1, 1), then the value of $-41[y(2)]^3$ is _____.

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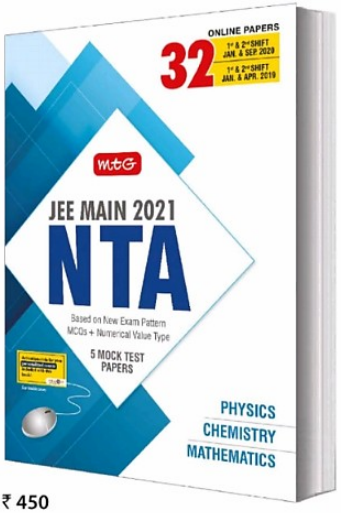

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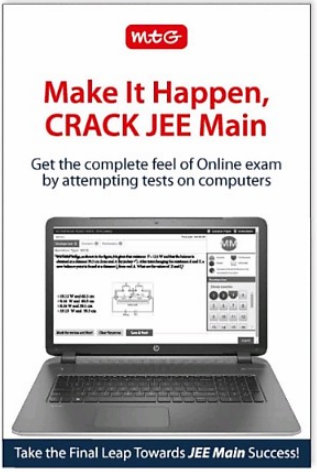
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